UNIT 2 : PRACTICE PROBLEMS

① $y = \ln\left(\frac{e^x}{e^x - 10}\right)$

$y = \ln e^x - \ln(e^x - 10)$

$y = x - \ln(e^x - 10)$

$y' = 1 - \frac{1}{(e^x - 10)} \cdot e^x$

$y' = \frac{e^x - 10}{e^x - 10} - \frac{e^x}{e^x - 10} = \frac{-10}{e^x - 10}$

$\boxed{y' = \frac{10}{10 - e^x}}$

② $y = (2x)5^{(x^2)}$

$y' = (2x)(5^{x^2})(\ln 5)(2x) + (5^{x^2})(2)$

$\boxed{y' = 2(5^{x^2})\left[2x^2(\ln 5) + 1\right]}$

③ $s(t) = t^2 - 6t - 4 \quad [0, 4]$

$v(t) = 2t - 6$

$v = 0 \quad 2t - 6 = 0$

$t = 3$

$s(0) = -4$

$s(3) = 9 - 18 - 4 = -13$

$s(4) = 16 - 24 - 4 = -12$

9

1

DISTANCE: $9 + 1 \boxed{= 10}$

④ $x^2 + xy + y^2 = 27$

$2x + x\,dy/dx + y + 2y\,dy/dx = 0$

$x\,dy/dx + 2y\,dy/dx = -2x - y$

$\frac{dy}{dx}(x + 2y) = -2x - y$

$\boxed{\frac{dy}{dx} = \frac{-2x - y}{x + 2y} \text{ or } -\frac{2x + y}{x + 2y}}$

5 VERTICAL TANG $\frac{dy}{dx} = \text{und}$ $\frac{\#}{0}$

y POSITIVE

$y = 3$

$x + 2y = 0$

$x = -2y$

$x^2 + xy + y^2 = 27$

$(-2y)^2 + (-2y)(y) + y^2 = 27$

$4y^2 - 2y^2 + y^2 = 27$

$3y^2 = 27$

$y = \pm 3$

$x^2 + 3x + 9 = 27$

$x^2 + 3x - 18 = 0$

$(x+6)(x-3) = 0$

$x = -6 \quad x = 3$

$\boxed{\text{PTS: } (-6, 3) \text{ and } (3, 3)}$

6 $y = \log_3\left(\frac{2}{x}\right)$ $(2x^{-1})$

$\frac{dy}{dx} = \frac{1}{\left(\frac{2}{x}\right)} \cdot \frac{1}{\ln 3} \cdot (-2x^{-2})$

$\frac{dy}{dx} = \frac{x}{2} \cdot \frac{1}{\ln 3} \cdot \frac{-2}{x^2} = \boxed{\frac{-1}{x \ln 3}}$

7 $y^3 - xy^2 = 4$ $y = 2$ $(1, 2)$

$y(2)$ $8 - 4x = 4$

$-4x = -4$

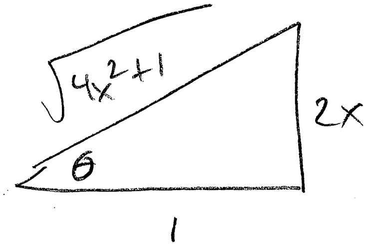
$x = 1$

$3y^2 \, dy/dx - x(2y) \, dy/dx - y^2 = 0 \Big|_{(1,2)}$

$12 \, dy/dx - 4 \, dy/dx - 4 = 0$

$8 \, dy/dx = 4$ $\boxed{\frac{dy}{dx} = \frac{1}{2}}$

⑧ $\frac{d}{dx}\left[\sin(\underbrace{\tan^{-1}(2x)}_{\theta})\right]$



$$\frac{d}{dx}[\sin\theta] = \frac{d}{dx}\left[\frac{2x}{\sqrt{4x^2+1}}\right]$$

$$= \frac{(4x^2+1)^{1/2}(2) - (\cancel{2}x)\left[\cancel{\tfrac{1}{2}}(4x^2+1)^{-1/2}\right](8x)}{(4x^2+1)^{1}}$$

$$= \frac{\frac{2(4x^2+1)}{(4x^2+1)^{1/2}} - \frac{8x^2}{(4x^2+1)^{1/2}}}{(4x^2+1)} = \frac{\frac{2}{(4x^2+1)^{1/2}}}{(4x^2+1)} = \boxed{\frac{2}{(4x^2+1)^{3/2}}}$$

⑨ $y = \ln(x^2+y^2)$ @ $(1,0)$

$$\frac{dy}{dx} = \frac{1}{x^2+y^2}\cdot\left(2x + 2y\,\frac{dy}{dx}\right)\Bigg|_{(1,0)}$$

$$\frac{dy}{dx} = \frac{2+0}{1} = \boxed{2}$$

⑫ $y = \underline{e^{-x}}\,\underline{\ln x}$ $\quad x=1$

$$\frac{dy}{dx} = (e^{-1})(1) + \underbrace{\ln(1)}_{0}\cancel{(-e^{-1})}$$

$$\frac{dy}{dx} = (e^{-x})(1/x) + \ln x(-e^{-x})\Big|_{x=1} \quad \boxed{\frac{dy}{dx} = \frac{1}{e}}$$

(13) $y = 2x e^{-x}$

Horz. $y' = 0$

$y' = (2x)(-e^{-x}) + e^{-x}(2)$

$-2xe^{-x} + 2e^{-x} = 0$

$-2e^{-x}(x-1) = 0$

$\boxed{x=1}$ -B

(14) $y = [\sin x]^{\ln x}$

$\ln y = \ln x\,[\ln(\sin x)]$

$\frac{1}{y} \cdot \frac{dy}{dt} = \ln x \left[\frac{\cos x}{\sin x}\right] + \ln(\sin x)\left(\frac{1}{x}\right)$

$\boxed{\frac{dy}{dx} = \left[\ln x \cot x + \frac{\ln(\sin x)}{x}\right]\left[\sin x^{\ln x}\right]}$

(15) $g'(4)$

$g(4, -)$

$f(-, 4)$

$f(2) = 4$

$g'(4) = \frac{1}{f'(2)} = \boxed{\frac{1}{5}}$

(16)

A) $\frac{2f(x)}{g(x)+1}$

$x = 0$

$\frac{[g(x)+1]\,2f'(x) - 2f(x)\,g'(x)}{[g(x)+1]^2}$

$= \frac{(2)(2)(0) - 2(7)(-3)}{(2)^2} = \frac{42}{4} = \boxed{\frac{21}{2}}$

16 B) $g[f(x)+x] \quad x=1$

$g'[f(x)+x]\cdot(f'(x)+1)$

$g'(-3+1)(-3+1) = g'(-2)(-2) = (-1)(-2) = \boxed{2}$

(17) $y = x^2 \sin^{-1}(1-2x)$

$$\frac{dy}{dx} = x^2\left[\frac{-2}{\sqrt{1-(1-2x)^2}}\right] + \sin^{-1}(1-2x)\cdot(2x)$$

$$\frac{dy}{dx} = \frac{-2x^2}{\sqrt{1-1+4x-4x^2}} + 2x\sin^{-1}(1-2x) = \frac{-2x^2}{\sqrt{4x-4x^2}} + 2x\sin^{-1}(1-2x)$$

$$\frac{dy}{dx} = \frac{-\cancel{2}x^2}{\cancel{2}\sqrt{x-x^2}} + 2x\sin^{-1}(1-2x) = \boxed{\frac{-x^2}{\sqrt{x-x^2}} + 2x\sin^{-1}(1-2x)}$$

(18) $f(x)$ IS ONE-TO-ONE $\Rightarrow f(x)$ HAS INVERSE

$f(a)=b \quad f(a,b)$

$\boxed{E}$ NONE

(19) $\lim_{x\to\infty} \frac{\ln x}{2x} \overset{LH}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{2} = \lim_{x\to\infty} \frac{1}{2x} = \boxed{0}$

(20) $f(x) = x^2 - 2x + 3e^x$ $\quad f(0,3)$ $g(3,0)$

$g'(3) = \frac{1}{f'(0)} = \boxed{1}$ $\quad f'(x) = 2x - 2 + 3e^x$

$f'(0) = -2 + 3 = 1$

(21) THE PARTICLE IS MOVING BACKWARDS WHEN $v(t) < 0$, ON THE INTERVALS $(1,5)$

(22) $a(t) = 0$ ON INTERVAL $(2,3)$

(23) SPEED IS INCREASING $(1,2)$ b/c $v<0$ & $a<0$

$(5,6)$ b/c $v>0$ & $a>0$

(24) THE PARTICLE CHANGES DIRECTIONS 2 TIMES.

(25) $\lim_{h\to 0} \frac{3^h - 1}{2h} \overset{LH}{=} \lim_{h\to 0} \frac{(3^h)(\ln 3)}{2} = \boxed{\frac{\ln 3}{2}}$ $\boxed{D}$

(26) $\lim_{h\to 0} \frac{2\ln(e^2+h) - 2\ln e^2}{h}$ $\quad f(x) = 2\ln x$

$f'(x) = \frac{2}{x}$ $\quad \boxed{f'(e^2) = \frac{2}{e^2}}$

(27) AVG VEL $= \frac{s(6) - s(4)}{6-4} = \frac{12-8}{2} = \boxed{2 \text{ ft/s}}$

$[4,6]$

(28) $v(1) \approx \frac{s(1.2) - s(0)}{1.2 - 0} = \frac{5-3}{1.2} = \frac{2}{1.2} = \frac{2}{\frac{6}{5}}$

$= \frac{10}{6} = \boxed{5/3 \text{ ft/s}}$

(29) DISPLACEMENT: $s(6) - s(0) = 12 - 3 = \boxed{9}$

(30) $1 + \ln xy = e^{x-y}$ $(1,1)$

$1 + \ln x + \ln y = e^{x-y}$

$\frac{1}{x} + \left(\frac{1}{y}\right) dy/dx = e^{x-y}(1 - dy/dx) \Big|_{(1,1)}$

$1 + dy/dx = 1 - dy/dx$

$2\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 0$

$\boxed{y = 1}$