AP Calculus – Unit 2
Advanced Differentiation Practice Problems

(1) If $y = \ln\left(\frac{e^x}{e^x-10}\right)$, then $\frac{dy}{dx} =$

(A) $x - \frac{e^x}{e^x-10}$ (B) $-\frac{1}{e^x}$ (C) $\frac{10}{10-e^x}$ (D) 0 (E) $\frac{e^x-20}{e^x-10}$

(2) Find $\frac{dy}{dx}$ of $y = (2x)5^{x^2}$

(3) A particle is moving along a linear path. The path of the particle is modeled by $s(t) = t^2 - 6t - 4$, where $t > 0$. Determine the total distance the particle travels over the first four seconds.

Use the following information to answer questions 4 and 5. Consider the curve defined by $x^2 + xy + y^2 = 27$

(4) Write an expression for the slope of the curve at any point (x,y).

(5) Find the points on the curve where the lines tangent to the curve are vertical and the y value is positive.

(6) Find $\frac{dy}{dx}$ of $y = \log_3\left(\frac{2}{x}\right)$

(7) Find the slope of the curve $y^3 - xy^2 = 4$ at the point where $y = 2$.

(8) Find $\frac{d}{dx}[\sin(\tan^{-1}[2x])]$

(A) $\frac{2(4x^2+1)^{3/2}-8x^2}{(4x^2+1)^{3/2}}$

(B) $\frac{-2}{(4x^2+1)^{3/2}}$

(C) $\frac{2}{1+4x^2}$

(D) $\frac{(4x^2+1)^{3/2}-4x^2}{(4x^2+1)^{3/2}}$

(E) $\frac{2}{(4x^2+1)^{3/2}}$

(9) Find the value of $\frac{dy}{dx}$ if y = $\ln(x^2 + y^2)$ at (1,0).

(12) If $y = e^{-x} \ln x$, what does $\frac{dy}{dx}$ equal when x = 1?

(13) The tangent to the curve $y = 2xe^{-x}$is horizontal when x =

(A) -2 (B) 1 (C) -1 (D) $\frac{1}{e}$ (E) None of the Above

(14) Differentiate: $y = [\sin x]^{\ln x}$

(15) If f(g(x)) = x = g(f(x)), use the chart below to determine g'(4).

	F(x)	G(x)	F'(x)
X = 1	1	-4	-4
X = 2	4	-2	5
X = 3	2	8	7
X = 4	-3	2	½

(16) Using the table to the right, determine the derivative of each expression.

(a) $\frac{2f(x)}{g(x)+1}$ at x = 0

(b) $g[f(x) + x]$ at x = 1

	F(x)	G(x)	F'(x)	G'(x)
X = -2	1	-4	5	-1
X = 0	7	1	0	-3
X = 1	-3	3	-3	3

(17) Find $\frac{dy}{dx}$: $y = x^2 \sin^{-1}(1 - 2x)$

(18) Assume that f(x) is one-to-one and f(a) = b. Which of the statements is false:

(A) $f^{-1}(x)$ will have a reciprocal slope of f(x) at corresponding points.

(B) f^{-1} (b) = a

(C) $(f^{-1})'(b) = \frac{1}{f\prime(a)}$

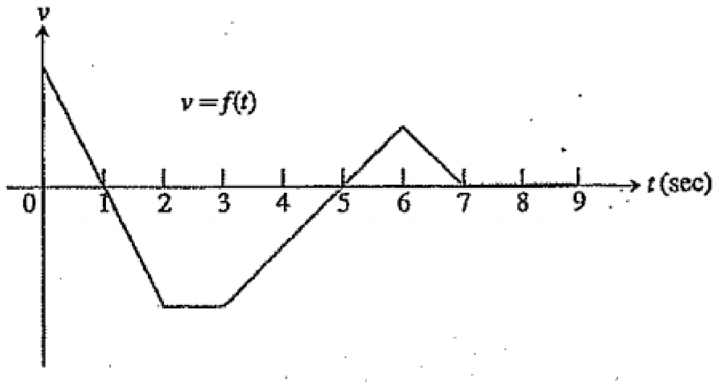
(D) The graphs of f(x) and f^{-1} (x) are symmetrical over the line y = x

(E) None of the Above

(19) $\lim_{x \to \infty} \frac{\ln x}{2x}$

(20) Let f be the function defined by $f(x) = x^2 - 2x + 3e^x$. If $g(x) = f^{-1}(x)$ for all x and the point (0,3) is on the graph of f, what is the value of $g'(3)$?

The graph below shows the velocity $v = f(t)$ of a particle, in ft/sec, moving along a horizontal line $0 \leq t \leq 7$ seconds. Use the graph to answer questions 21 – 24.



(21) On what open intervals or at what time(s) $0 < t < 7$ is the particle moving backwards? Justify.

(22) On what open intervals or at what time(s) $0 < t < 7$ is the particle's acceleration zero?

(23) On what open intervals or at what time(s) $0 < t < 7$ is the particle's speed increasing? Justify.

(24) On what open intervals or at what time(s) $0 < t < 7$ how many times the particle change directions?

(25) Evaluate the limit $\lim_{h \to 0} \frac{3^h - 1}{2h}$.

(a) 1 (b) 0 (c) ∞ (d) $\frac{\ln 3}{2}$ (e) DNE

(26) Evaluate the limit $\lim_{h \to 0} \frac{2\ln(e^2 + h) - 2\ln e^2}{h}$.

The table below shows the position s of a particle moving continuously along a line for various times over the interval [0,6]. Use the table to answer questions 27 – 29.

t secs	0	1.2	2.5	4.0	5.1	6
s(t) feet	3	5	1	8	10	12

(27) What is the particle's average velocity over the interval [4,6]? Show the work that leads to your answer.

(28) Estimate the velocity of the particle at $t = 1$ second. Show the work that leads to your answer.

(29) What is the displacement of the particle over the interval [0,6]?

(30) Write the equation of the tangent to the graph $1 + \ln xy = e^{x-y}$ at the point (1,1)