(1) Find the absolute minimum and maximum values of $f(x) = 3x^{2/3} - 2x + 1$ on the interval [-1,8].

Consider the curve given by the equation $y^3 + 3x^2y + 13 = 0$ to answe questions 2 and 3.

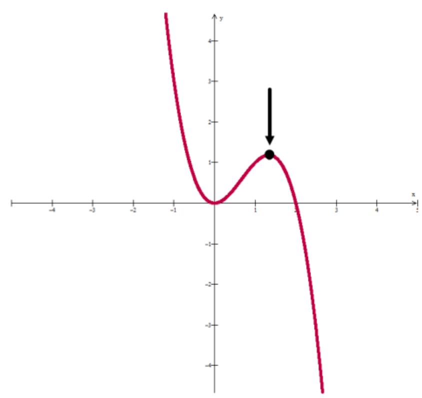
(2) Find dy/dx.

(3) Write an equation of the tangent line at the point (2,-1).

(4) Let *g(x)* be a function that is differentiable for all *x*. Suppose that *g*(-2) = 1 and -2 ≤ *g'(x)* ≤ 5 for all value of *x*. Which the following cannot be a value for *g*(1)?

(A) -6 (B) -4 (C) 0 (D) 4 (E) 16

(5) Use the graph to the right of *f(x)* to determine which of the following statements is true if the point is at *x* = *c*.



(A) $f(c) < f'(c) < f''(c)$

(B) $f'(c) < f(c) < f''(c)$

(C) $f''(c) < f'(c) < f(c)$

(D) $f(c) < f''(c) < f'(c)$

(E) $f''(c) < f(c) < f'(c)$

(6) The total number of all relative extrema of the function *F* whose derivative is $F'(x) = x(x-3)^2(x+2)^3$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) None of These

(7) Which of the following functions fails to satisfy the conclusion of the Mean Value Theorem on the given interval?

(A) $3x^{2/3} - 1$; [1,2] (B) $|3x - 2|$; [1,2]

(C) $4x^3 - 2x + 3$; [0,2] (D) $\sqrt{x-2}$; [3,6]

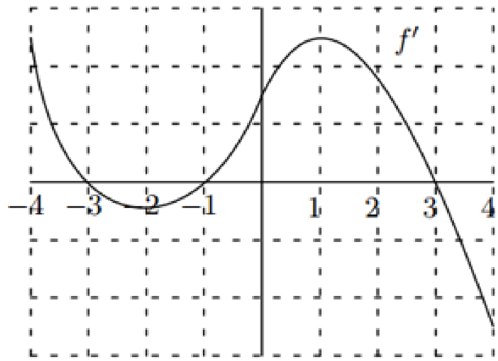
(E) None of These

Use the following information to answer questions 8 and 9. Let *f* be continuous function on [-3,3] whose first and second derivatives have the following signs and values.

x	$-3 < x < -1$	-1	$-1 < x < 0$	0	$0 < x < 1$	1	$1 < x < 3$
$f'(x)$	positive	0	negative	negative	negative	0	negative
$f''(x)$	negative	negative	negative	0	positive	0	negative

(8) What are the *x*-coordinates of the relative extrema of *f* on [-3,3]? Label each answer as a relative minimum or relative maximum.

(9) What are the *x* –coordinates of the points of inflection of *f* on [-3,3]?

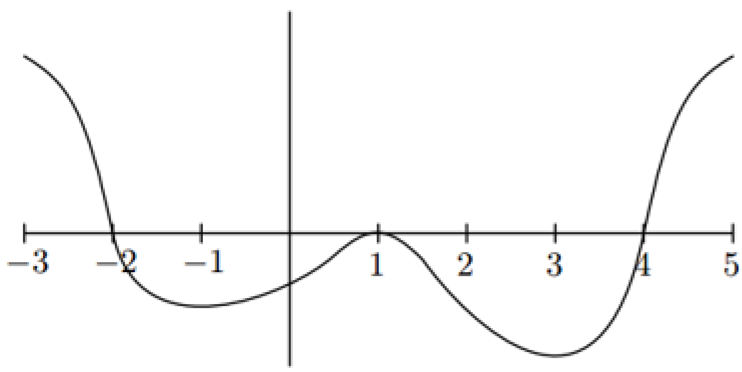


(10) The graph of the derivative of a function *f* is shown above. Which of the following are true about the original *f*?

I. *f* is increasing on the interval (-2,1).
II. *f* is continuous at $x = 0$.
III. *f* has an inflection point at $x = -2$.

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III

Use the information to answer questions 11 and 12. The figure below shows the graph of f', the derivative of a function *f*. The domain of *f* is the set of all real numbers *x* such that $-3 < x < 5$. The graph has zeros at $x = -2$, $x = 1$, and $x = 4$ and horizontal tangent lines at $x = -1$, $x = 1$, and $x = 3$.



(11) For what value(s) of *x* does *f* have relative maximum? Justify your answer.

(12) On what intervals is the graph of *f* concave upward? Justify your answer.

Use the following information to answer questions 13-14. Consider the function *f* defined by $f(x) = e^x \cos x$ with domain $[0, 2\pi]$

(13) Find the intervals on which *f* is increasing.

(14)Find the *x*-coordinate of each point of inflection of graph of *f*.

(15) If *P(x)* is continuous in [*k*,*m*] and differentiable in (*k*,*m*), then the Mean Value Theorem states that there is a point a between k and m such that

(A) $k - m = P(k) - P(m)$

(B) $\frac{P(m)-P(k)}{m-k} = P'(a)$

(C) $\frac{m-k}{P(m)-P(k)} = a$

(D) $\frac{m-k}{P(m)-P(k)} = P'(a)$

(E) None of These

x	f(x)	f ’(x)	f ’’(x)
-1	-2	4	0
1	4	0	5
4	6	-2	-1
6	0	3	4

(16) Assuming *f(x)* is continuous and twice differentiable for all x, Use the table above to determine any x-values for relative maximum and relative minimum. Justify your answer.

(17) Find a value *x* = *c* that satisfies the Mean Value Theorem for the function $y = x^{1/2} - x$ on the interval [0,4]. If the Mean Value Theorem does not apply, state the reason why.

(18) What is the minimum value of the slope of the curve $y = x^5 + x^3 - 2x$?

<u>STATION ONE: Analyzing graphs and tables.</u>

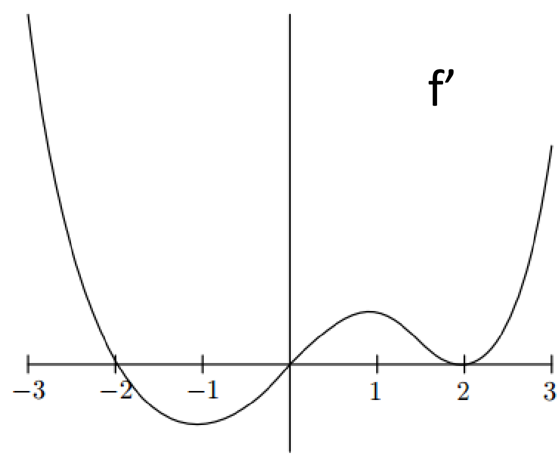
(19) The graph below is the derivative graph of *f*. Answer the following questions. Be sure to justify your answers.

A. Find all x-values where *f* attains a relative extrema.

B. Find all x-values where *f* is increasing.

C. Find all x-values the where *f* is concave up.

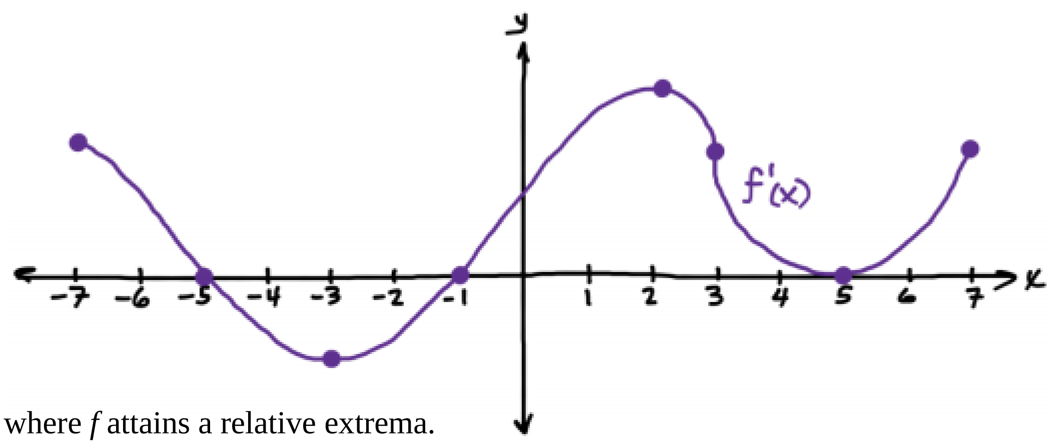
D. Find all x-values where *f* attains a point of inflection.



(20) Selected values for a twice-differentiable function f(x), continuous on, $0 \leq x \leq 5$ is given below along with selected values of f '(x), and f"(x). Determine if any x-values are relative max/min. Justify your answer.

x	f(x)	f '(x)	f "(x)
0	4	2	1
2	2	-2	0
5	1	0	-2

(21) The figure below shows the graph of *f* ', the derivative of the *f*, for $-7 \leq x \leq 7$. The graph of *f* ' has horizontal tangent lines at x = -3, x = 2 and x = 5. Answer the questions below. Justify your answers.



A. Find all x-values where *f* attains a relative extrema.

B. Find all x-values where *f*" < 0.

C. Find all x-values where *f* attains a point of inflection.

(22) Determine the intervals where $f(x) = (x-3)^{\frac{4}{5}}(x+1)^{\frac{1}{5}}$ is increasing. Justify.

(23) The derivative of a function *f* is given for all x by $f'(x) = (2x^2 + 4x - 16)(1 + g^2(x))$ where g is some unspecified function. At which value(s) of x will *f* have a local maximum? Justify.

(24) Find the absolute maximum and minimum values of $f(x) = \sin^2 x + \cos x$ on the interval $[0,2\pi]$.

(25) Find all relative extrema for the function $f(x) = x + \frac{4}{x}$. Justify.

(26) Let f be the function given by $f(x) = x^3 - 5x^2 + 3x + k$, where k is a constant.

A. On what intervals is the graph of *f* concave downward?

B. Find the value of k for which *f* has 11 as its relative minimum.

(27) A function *f* is continuous on the closed interval [-3,3] such that $f(-3) = 4$ and $f(3) = 1$. The function *f'* and *f''* have the properties given in the table below.

x	$-3 < x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$1 < x < 3$
$f'(x)$	positive	fails to exist	negative	0	negative
$f''(x)$	positive	fails to exist	positive	0	negative

A. What are the x-coordinates of all points of inflection of *f* on the given interval? Justify your answer.

B. Sketch a graph of *f* that satisfies the properties listed above.

(28) Find all points of inflection of the graph $f(x) = \frac{x^2}{x^2+3}$.

(29) Find the open intervals of the graph $f(x) = x \ln x$ is concave upward and concave downward. Justify your answer.

For problems #30 and 31, verify that MVT can be applied and then find the value of "c" that is guaranteed by the MVT.

(30) $f(x) = 3x^{\frac{2}{3}} - 2x$ on [0,1]

(31) $G(x) = (x-1)^3$ on [-1,2]

(32) If $f(x) = 2x^3 - 6x$, at which point on the interval $0 \leq x \leq \sqrt{3}$, if any, is the tangent to the curve parallel to the secant line on the interval?

(33) Which of the following functions below satisfy the hypothesis of the MVT?

I. $f(x) = \frac{1}{x+1}$ on [0,2]

II. $f(x) = x^{\frac{1}{3}}$ on [0,1]

III. $f(x) = |x|$ on [-1,1]

A. I only B. I and II only C. I and III only D. II only E. II and III only