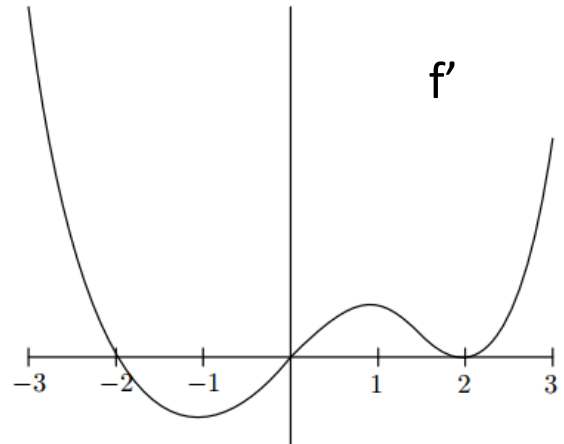


STATION ONE: Analyzing graphs and tables.

1. The graph below is the derivative graph of f . Answer the following questions. Be sure to justify your answers.

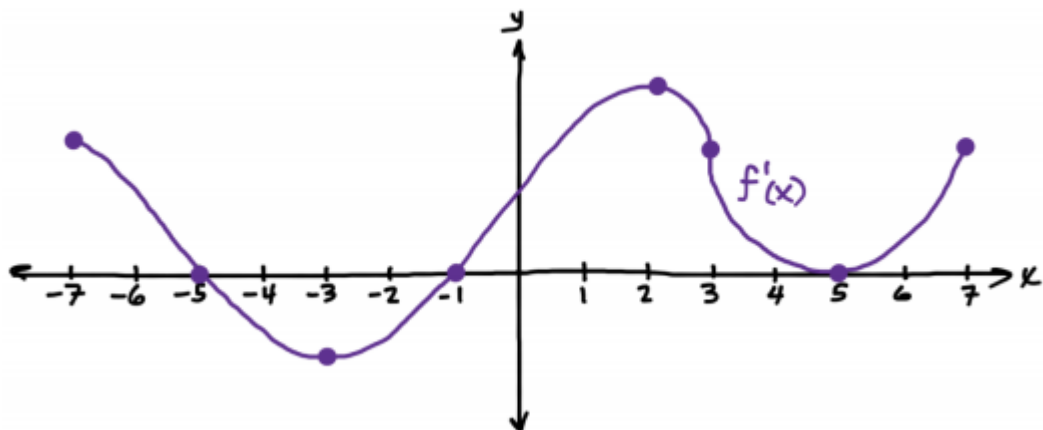
- A. Find all x -values where f attains a relative extrema.
- B. Find all x -values where f is increasing.
- C. Find all x -values the where f is concave up.
- D. Find all x -values where f attains a point of inflection.



2. Selected values for a twice-differentiable function $f(x)$, continuous on, $0 \leq x \leq 7$ is given below along with selected values of $f'(x)$, and $f''(x)$. Determine if any x -values are relative max/min. Justify your answer.

x	$f(x)$	$f'(x)$	$f''(x)$
0	4	2	1
2	2	-2	0
5	1	0	-2

3. The figure below shows the graph of f' , the derivative of the f , for $-7 \leq x \leq 7$. The graph of f' has horizontal tangent lines at $x = -3$, $x = 2$ and $x = 5$. Answer the questions below. Justify your answers.



- A. Find all x -values where f attains a relative extrema.
- B. Find all x -values where $f'' < 0$.
- C. Find all x -values where f attains a point of inflection.

STATION TWO: First Derivative Test – increasing, decreasing, and finding extrema.

1. Determine the intervals where $f(x) = (x - 3)^{\frac{4}{5}}(x + 1)^{\frac{1}{5}}$ is increasing. Justify.
2. The derivative of a function f is given for all x by $f'(x) = (2x^2 + 4x - 16)(1 + g^2(x))$ where g is some unspecified function. At which value(s) of x will f have a local maximum? Justify.
3. Find the absolute maximum and minimum values of $f(x) = \sin^2 x + \cos x$ on the interval $[0, 2\pi]$.
4. Find all relative extrema for the function $f(x) = x + \frac{4}{x}$. Justify.

STATION THREE: Second Derivative Test – concavity and points of inflection.

1. Let f be the function given by $f(x) = x^3 - 5x^2 + 3x + k$, where k is a constant.

A. On what intervals is the graph of f concave downward?

B. Find the value of k for which f has 11 as its relative minimum.

2. A function f is continuous on the closed interval $[-3,3]$ such that $f(-3) = 4$ and $f(3) = 1$.

The function f' and f'' have the properties given in the table below.

x	$-3 < x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$1 < x < 3$
$f'(x)$	positive	fails to exist	negative	0	negative
$f''(x)$	positive	fails to exist	positive	0	negative

A. What are the x -coordinates of all points of inflection of f on the given interval? Justify your answer.

B. Sketch a graph of f that satisfies the properties listed above.

3. Find all points of inflection of the graph $f(x) = \frac{x^2}{x^2+3}$.

4. Find the open intervals of the graph $f(x) = x \ln x$ is concave upward and concave downward. Justify your answer.

STATION FOUR: Mean Value Theorem – apply the MVT to solve problems.

For problems #1-2, verify that MVT can be applied and then find the value of “c” that is guaranteed by the MVT.

1. $f(x) = 3x^{\frac{2}{3}} - 2x$ on $[0,1]$

2. $G(x) = (x - 1)^3$ on $[-1,2]$

3. If $f(x) = 2x^3 - 6x$, at which point on the interval $0 \leq x \leq \sqrt{3}$, if any, is the tangent to the curve parallel to the secant line on the interval?

4. Which of the following functions below satisfy the hypothesis of the MVT?

I. $f(x) = \frac{1}{x+1}$ on $[0,2]$

II. $f(x) = x^{\frac{1}{3}}$ on $[0,1]$

III. $f(x) = |x|$ on $[-1,1]$

A. I only B. I and II only C. I and III only D. II only E. II and III only

Station 5: Related Rates

(1) Tom is walking towards a lamppost that is 22 feet tall. If Tobi is 6 feet tall and he is walking at a rate of 1 foot per second, how fast is his shadowing changing when he is 8 feet from the lamppost?

(2) Andrew is trying to break the record for largest rubber band ball. As of right now, his rubber band ball has a diameter of 28 centimeters across. If the rate of change in the volume is 500 cubic centimeters per month, how fast is the radius changing?

(3) A 14-foot ladder is leaning against the wall of a house. At the moment the ladder is 6 feet away from the house, the ladder is sliding down the wall at a rate of 2 feet per second.

(a) Find the rate at which the ladder is moving away from the base of the wall.

(b) What is the rate of change in the area created between the wall and the ladder?

(c) What is the rate in change in the angle created between the ladder and the ground?

(4) Wesley is drinking out of the coolest new water bottle which takes the shape of a cone that has an opening at the vertex. The base of the water bottle is 4 inches across and the height is 9 inches. When Wesley tips the bottle upside down to drink, he is consuming 4 cubic inches per second (he is really thirsty after studying so hard for Calculus). How fast is the radius of the water changing when the depth of the water is 4 inches?