$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$ $\cos(3x) = 1 - \frac{(3x)^2}{2!} + \frac{(3x)^4}{4!}$

$\frac{81}{4!} = \frac{81 \div 3}{24 \div 3}$

WARM UP

1. Which of the following coefficient of x^4 in the Maclaurin polynomial generated by cos(3x)?

(a) 27/8 (b) 9 (c) 1/24 (d) 0 (e) -27/8

2. Determine if the following converges/diverges. Justify.

Ratio Test or AST

$$\sum_{n=1}^{\infty} \frac{(-1)^n 3^n}{n2^n}$$

$a_n = \frac{3^n}{n2^n} = \frac{1}{n}\left(\frac{3}{2}\right)^n$

$\lim_{n\to\infty} \left(\frac{1}{n}\right)\left(\frac{3}{2}\right)^n \neq 0$ diverges by AST

Power Series: Day 1

Objective:

- Understand the definition of a power series.
- Find the radius and interval of convergence.
- Determine the endpoint of convergence of a power series

If x is a variable, then an infinite series of the form:

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \cdots a_n x^n + \cdots$$

is called a power series

$$\sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_1 (x-c) + a_2 (x-c)^2 + \cdots a_n (x-c)^n + \cdots$$

is a power series center at c, where c is a constant

A power series in x can be viewed as a function of x where the domain of f is the set of all x for which the power series converges.

Our job is to find what value of x the series converges.

For a power series centered at c, precisely <u>one</u> of the following is true:

(1) The series converges only at c (ALL power series converge at their centers!)

The radius is 0

The domain of f(x), also called the interval of convergence, would be a single point (the center).

(2) The series converges for all x (function and series are equal for all values)

The radius is ∞

The IOC $(-\infty, \infty)$. The series converges absolutely for all x.

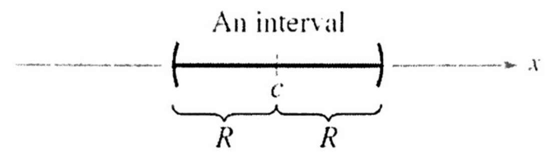
example: $\sin x = x - \frac{x^3}{3!} + \dots$, true for all x

(3) There exist a $R > 0$ such that the series converges for $|x-c| < R$ and diverges for $|x-c| > R$

*(no way a polynomial can approximate all terms of a rational)

R is called the radius of convergence of the power series

IOC - $[(c-R, c+R)]$



You need to check each endpoint independently to see if the converges/diverges.

Ex.1: Find the nth term for the power series f(x) = e^x, centered at zero and then find the radius and interval of convergence for the representative power series.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots + \frac{x^n}{n!} + \dots$$

$$= \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

n must start a zero

2 ways to represent a series

Ratio Test

$$\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n} \right| = \lim_{n\to\infty} \left| \frac{x}{n+1} \right| = 0 < 1$$

for all x element of the reals. $\forall x \in \mathbb{R}$

Radius $= \infty$

(IOC): interval of convergence

$(-\infty, \infty)$

Ex.2: Find the radius of convergence and the interval of convergence. Be sure to check the endpoints.

$$\sum_{n=0}^{\infty}\left(\frac{x}{3}\right)^n = \sum_{n=0}^{\infty}\frac{x^n}{3^n}$$

Centered at 0

$|x-0|<r$

$|x|<r$

① Ratio Test

$$\lim_{n\to\infty}\left|\frac{x^{n+1}}{3^{n+1}}\cdot\frac{3^n}{x^n}\right| = \lim_{n\to\infty}\left|\frac{x}{3}\right| = \left|\frac{x}{3}\right|<1$$

$3\cdot\frac{|x|}{3}<1\cdot 3$

② Test endpoints ☺

$x=3$

$\sum_{n=0}^{\infty}\left(\frac{3}{3}\right)^n$ diverges nth term test

$\lim_{n\to\infty}(1)^n=\infty$

$x=-3$

$\sum_{n=0}^{\infty}\left(\frac{-3}{3}\right)^n$ nth term test

$\lim_{n\to\infty}(-1)^n\neq 0$ diverge by oscillation

radius is 3 $|x|<3$ not sure which one.

I.o.C. $[(-3,3)]$

IOC $(-3,3)$

Ex.3: Find the radius of convergence and the interval of convergence. Be sure to check the endpoints.

centered at 5

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(x-5)^n}{n2^n}$$

① Ratio test

$$\lim_{n\to\infty}\left|\frac{(x-5)^{n+1}}{(n+1)2^{n+1}}\cdot\frac{n2^n}{(x-5)^n}\right| = \lim_{n\to\infty}\left|\frac{(x-5)n}{(n+1)(2)}\right| = \left|\frac{x-5}{2}\right|<1$$

$\left|\frac{x-5}{2}\right|<1$ $|x-5|<2$ radius is 2 IOC $[(3,7)]$

② test endpoints

$x=3$

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(-2)^n}{n2^n} = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}(-1)^n}{n} = \sum_{n=1}^{\infty}-\frac{1}{n}$$

diverges harmonic p-series

$x=7$

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(2)^n}{n(2)^n}$$

converges by AST

$\lim_{n\to\infty}\frac{1}{n}$ decreases

IOC $(3,7]$

Ex.4: Find the radius of convergence and the interval of convergence. Be sure to check the endpoints.

don't forget to distribute.

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$a_{n+1} = \frac{(-1)^{n+1} x^{2(n+1)+1}}{(2(n+1)+1)!}$$

① Ratio Test

$$\lim_{n\to\infty} \left| \frac{x^{2n+3}}{(2n+3)!} \cdot \frac{(2n+1)!}{x^{2n+1}} \right| = \lim_{n\to\infty} \left| \frac{x^2}{(2n+3)(2n+2)} \right| = 0 < 1$$

$R = \infty$

IOC : $(-\infty, \infty)$

Ex.5: Find the radius of convergence and the interval of convergence. Be sure to check the endpoints.

$$\sum_{n=0}^{\infty} n!(x-3)^n$$

$c = 3$ centered at 3

$$\lim_{n\to\infty} \left| \frac{(n+1)!\,(x-3)^{n+1}}{n!\,(x-3)^n} \right| = \lim_{n\to\infty} \left| (n+1)(n-3) \right| = \infty > 1$$

$R = 0$

Series converges at $x = 3$

or $[3]$

CLASSWORK - Find the radius and interval of convergence of the power series.

$$\sum_{n=1}^{\infty} \frac{(4x)^n}{n^2} \qquad \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} \qquad \sum_{n=0}^{\infty} \left(\frac{x}{4}\right)^n \qquad \sum_{n=0}^{\infty} (2n)!\left(\frac{x}{3}\right)^n$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-1)^{n+1}}{n+1} \qquad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-4)^n}{n9^n}$$