Particle Motion Worksheet

If $x(t)$ represents the position of a particle along the x-axis at any time, t, fill in the blanks in the statements below with the best answer so that they become true facts (not opinions).

(a) "Initially" means when t $= 0$.

(b) "At the origin" means $x + y$ $= 0$.

(c) "At rest" means velocity $= 0$.

(d) If the velocity of the particle is positive, then the particle is moving to the right.

(e) If the velocity of the particle is negative then the particle is moving to the left.

(f) To find average velocity over a time interval, divide the change in position by the change in time.

(g) Instantaneous velocity is the velocity at a single moment (instant) in time.

(h) If the acceleration of the particle is positive, then the velocity is increasing.

(i) If the acceleration of the particle is negative, then the velocity is decreasing.

(j) In order for a particle to change directions, the velocity must change signs.

(k) One way to determine total distance traveled over a time interval, when given the position function or graph, is to find the sum of the absolute values of the differences in position between all resting points.

If the position of a particle along a horizontal line is given by $x(t) = x^2 + x - 6$ for $0 \le t \le 3$

(a) Sketch the graph of the particle's position on the given interval.

(b) What is the particle's displacement on the given interval? Show the work that leads to your answer.

$x(3) = 9 + (3) - 6 = 12 - 6 = 6$

$x(0) = -6$ 12 units

(c) Find the total distance traveled by the particle on the given interval. Show the work that leads to your answer.

$x'(t) = v(t) = 2x + 1$

$0 = 2x + 1$

$-1 \quad -1$

$\frac{-1}{2} = \frac{2x}{2}$

not in interval

total distance = displacement

12 units

The data in the table below gives selected values for the velocity, in meters/minute, of a particle moving along the x-axis. The velocity v is a differentiable function of time t.

Time t (min)	0	2	5	6	8	12
Velocity $v(t)$ (meters/min)	−3	2	3	5	7	5

(a) At $t = 0$, is the particle moving to the right or to the left? Justify.

particle is moving to the left because $v(0)$ is negative.

(b) Is there a time during the time interval $0 \le t \le 12$ minutes when the particle is at rest? Explain your answer.

the particle is at rest between 0 and 2 seconds because the velocity changes signs (IVT)

(c) Use data from the table to find an approximation for $v'(10)$ and explain the meaning of $v'(10)$ in terms of the motion of the particle. Show the computations that lead to your answer, and indicate units of measure.

$v'(10) = \frac{v(12) - v(8)}{12 - 8} = \frac{5 - 7}{4} = \frac{-2}{4} = \frac{-1}{2}$

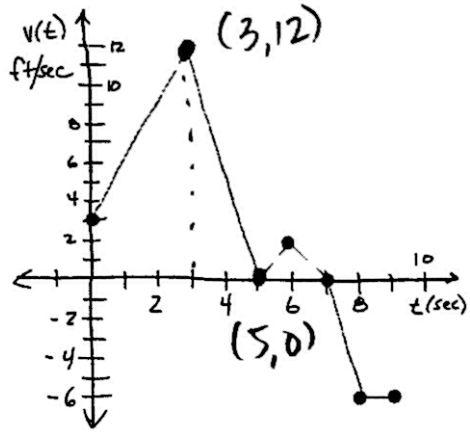
$-\frac{1}{2}$ meters/min^2

(d) Find the average acceleration of the particle for $8 \le t \le 12$ min. Explain what this number means in terms of the particle's velocity on that interval.

$\frac{v(12) - v(8)}{12 - 8} = \frac{5 - 7}{4} = -\frac{1}{2}$ the particle is slowing down

(e) Let $a(t)$ denote the acceleration of the particle at time t, such that $v'(t) = a(t)$. Is there guaranteed to be a time $t = c$ in the interval $0 \le t \le 12$ such that $a(c) = 0$? Justify your answer. $a < 0$ and $v > 0$

yes, $v(6) = 5$ and $v(12) = 5$, therefore there must be a c between $t = 6$ and 12 where the $a(c) = 0$ by the mean value theorem.



The graph above represents the velocity v, in feet per second, of a particle moving along the x-axis over the time interval for $0 \le t \le 9$ seconds.

(a) At $t = 4$ seconds, is the particle moving to the right or left? Justify.

moving to the right, $v(4) > 0$

(b) At what time(s) is the particle at rest? Justify.

$t = 5$, $t = 7$ because $v = 0$

(c) At what time(s) does the particle change direction? Justify.

$t = 7$ because velocity changes from positive to negative.

(d) On what open intervals $0 < t < 9$ is the particle moving left? Justify.

$(7,9)$ because $v(t) < 0$

(e) What is the acceleration of the particle at $t = 4$ seconds? Show the work that leads to your answer.

$$\frac{12-0}{3-5} = \frac{12}{-2} = -6 \text{ ft/sec}^2$$

(f) On what open intervals $0 < t < 9$ is the acceleration of the particle positive? Justify.

$(0,3) \cup (5,6)$ because velocity is increasing

(g) What is the average acceleration of the particle over the interval $t \in [3,6]$ seconds? Show the computations that lead to your answer, and indicate units of measure. $(3,12)$ $(6,1)$

$$\frac{v(6)-v(3)}{6-3} = \frac{2-12}{6-3} = \frac{-10}{3} \text{ ft/sec}^2$$

(h) On what open intervals $0 < t < 9$ is the **speed** of the particle decreasing? Justify.

speed decreases when $v > 0$ + $a < 0$ @ $(3,5) \cup (6,7)$

(i) Without knowing the initial position of the particle, is it still possible to determine the time at which the particle is farthest right for $0 \le t \le 9$? If not, explain. If so, find this value of t, and explain.

yes @ $t = 7$ because particle moves to right from $t = 0$ to $t = 5$ then from $t = 5$ to $t = 7$ it is moving to the left.

A particle moves along the x-axis so that at time $t \geq 0$ hours, its position is given by $x(t) = t^3 - 3t^2 - 9t + 2$ miles.

(a) At $t = 0$, is the particle moving to the right or to the left? Justify.

$x'(t) = 3t^2 - 6t - 9$ moving to the left because $v(0) < 0$

$x'(0) = -9$

(b) At what time(s) does the particle change directions. Justify.

$v(t) = 3(t^2 - 2t - 3)$ particle changes direction at $t = 3$ because $v(t)$ changes signs at $t = 3$

$0 = 3(t-3)(t+1)$

$t = 3, \cancel{t = -1}$

(c) At $t = 1/2$, is the velocity of the particle increasing or decreasing? Explain your answer.

$a(t) = 6t - 6$ decreasing because $a(\frac{1}{2}) < 0$

$a(\frac{1}{2}) = 6(\frac{1}{2}) - 6 = 3 - 6 = -3$

(d) At $t = 1/2$, is the speed of the particle increasing or decreasing? Explain your answer.

$v(\frac{1}{2}) = 3(\frac{1}{4}) - 6(\frac{1}{2}) - 9 = \frac{3}{4} - 3 - 9 < 0$

$v(\frac{1}{2}) < 0$ and $a(\frac{1}{2}) < 0$ $\therefore$ particle is speeding up

(e) Find all values of t for which the particle is moving to the left.

$v(t) = 3(t-3)(t+1)$ (−) 0 (+) 3 the particle is moving left at $(0, 3)$

$0 = 3(t-3)(t+1)$

(f) What is the particle's acceleration at $t = \frac{1}{3}$? Explain, with units, the meaning of your answer in terms of the particle's velocity.

$a(\frac{1}{3}) = 6(\frac{1}{3}) - 6 = 2 - 6 = -4$ units

Fill in the blanks so that each statement below is true.

(a) If velocity is negative and acceleration is positive, then speed is decreasing.

(b) If velocity is positive and speed is decreasing, then acceleration is negative.

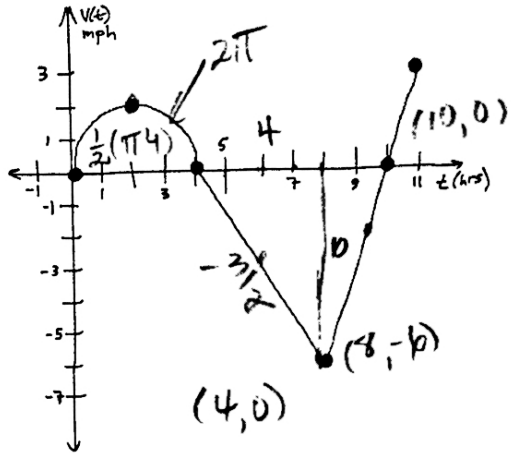
(c) If velocity is positive and decreasing, then speed is decreasing.

$v > 0$

(d) If speed is increasing and acceleration is negative, then velocity is negative.

(e) If velocity is negative and increasing, then speed is decreasing.

(f) If the particle is moving to the left and speed is decreasing, then acceleration is positive.



$\frac{-6-0}{8-4} = \frac{-6}{4} = -\frac{3}{2}$

$\frac{0-(-6)}{10-8} = \frac{6}{2} = 3$

The graph above shows the velocity, $v(t)$, in miles per hour of a particle moving along the x-axis for $0 \le t \le 11$ hours. It consists of a semi circle and two line segments. Use the graph and your knowledge of motion to answer the following questions.

(a) At what time, $0 \le t \le 11$ hours, is the speed of the particle the greatest?

$t = 8$ hrs

(b) At which of the times, $t = 2$, $t = 6$, or $t = 9$ hours, is the acceleration of the particle greatest? Justify.

$t = 9$ because $a(9) = 3$, $a(6) = -\frac{3}{2}$, $a(0) = 0$

(c) Over what open time interval(s) $0 < t < 11$ hours is the particle moving to the left? Justify.

$(4, 10)$ because $v(t) < 0$

(d) Over what open time interval(s) $0 < t < 11$ hours is the velocity of the particle increasing? Justify.

$(0, 2) \cup (8, 11)$

(e) Over what open time interval(s) $0 \le t \le 11$ hours is the speed of the particle increasing? Justify.

$a > 0$, $v < 0$: $(0, 2)$; $a < 0$, $v < 0$: $(4, 8)$; $a > 0$, $v > 0$: $(10, 11)$

$(0, 2) \cup (4, 8) \cup (10, 11)$

(f) At what times on $0 < t < 11$ is the acceleration of the particle undefined?

$t = 4$, $t = 8$

(g) Find the area of the semicircle on the interval $0 \le t \le 4$ bounded by the curve and the x-axis, then find the area of the triangle on the interval $4 \le t \le 10$ bounded by the curve and the x-axis, and finally, find the area of the triangle on the interval $10 \le t \le 11$ bounded by the curve and the x-axis. If all of these areas were positive and added together, propose what quantity this might be in terms of the particle's movement on $0 \le t \le 11$ hours.

$2\pi + \left(\frac{1}{2}(4)(6)\right) + \left(\frac{1}{2}(6)(2)\right) + \left(\frac{1}{2} \cdot 3 \cdot 1\right)$

$2\pi + 12 + 6 + \frac{3}{2}$

$2\pi + 18 + \frac{3}{2} = \frac{4\pi}{2} + \frac{36}{2} + \frac{3}{2} = \frac{4\pi + 39}{2}$