Parametric and Polar Practice Problem
AP Calculus BC

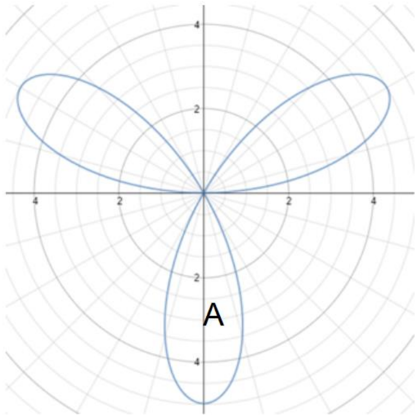
Part One: Calculator Inactive

(1) The position of a particle at any time $t \geq 0$ is given by $x(t) = \frac{3}{2}t^2 - 4$ and $y(t) = t^3 - 1$.

(a) Find the total distance traveled by the particle from $t = 0$ to $t = 4$. $\mathbf{17^{\frac{3}{2}} - 1}$

(b) Find $\frac{dy}{dx}$ as a function of x. $\frac{dy}{dx} = \left[\frac{2}{3}(x+4)\right]^{\frac{1}{2}}$

(2) The graph pictured to the right is represent by the equation $r = 5\sin(3\theta)$. Which of the following represents the area set-up with the correct limits of the petal labeled "A"?



(A) $\frac{1}{2}\int_0^{\pi/3} r^2 d\theta$

(B) $\frac{1}{2}\int_0^{\pi} r^2 d\theta$

(C) $\frac{1}{2}\int_0^{2\pi/3} r^2 d\theta$

(D) $\frac{1}{2}\int_{\pi/3}^{2\pi/3} r^2 d\theta$

(E) $\frac{1}{2}\int_{4\pi/3}^{5\pi/3} r^2 d\theta$

(3) Find $\frac{dy}{dx}$ of $r = 4\sin\theta$ at $\theta = \frac{\pi}{3}$. $-\sqrt{3}$

(4) Write the expression for the area located inside the graph $r = 3\sin\theta$ and outside the graph $r = \sin\theta - 1$. Set-up the integral, but do not evaluate.

$$A = \frac{1}{2}\int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} (3\sin\theta)^2 - (\sin\theta - 1)^2 d\theta$$

(5) Given that $x(t) = \sqrt{t}$ and $y(t) = \ln(t^2 - 5)$, find $\frac{d^2y}{dx^2}$ at $t = 1$. -4

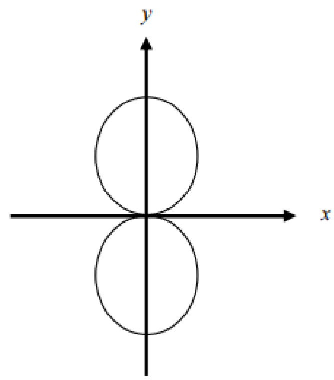
(6) The position of a particle moving in the xy-plane is given by the parametric equations $x = \frac{2}{3}t^3 - \frac{7}{2}t^2 + 3t$ and $y = \frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t - 1$. For what value(s) of t is the particle at rest? $t = 3$

(7) In the xy-plane, a particle moves along the parabola $y = x^2 - x$ with a constant speed of $2\sqrt{10}$ units per second. If $\frac{dx}{dt} > 0$, what is the value of $\frac{dy}{dt}$ when the particle is at the point $(2,2)$?

(A) $\frac{2}{3}$ (B) $\frac{2\sqrt{10}}{3}$ (C) 3 **(D) 6** (E) $6\sqrt{10}$

(8) Which of the following expressions gives the total area enclosed by the polar curve $r = \sin^2\theta$ shown in the figure below?

(A) $\frac{1}{2}\int_0^{\pi} \sin^2\theta d\theta$ (B) $\int_0^{\pi} \sin^2\theta d\theta$ (C) $\frac{1}{2}\int_0^{\pi} \sin^4\theta d\theta$ **(D) $\int_0^{\pi} \sin^4\theta d\theta$** (E) $2\int_0^{\pi} \sin^4\theta d\theta$



Part Two: Calculator Active

(1) The polar curve *r* is given by $r(\theta) = 3\theta + \sin\theta$, where $0 \le \theta \le 2\pi$.

(a) Find the area in the second quadrant enclosed by the coordinate axes and the graph of *r*. $\mathbf{47.513}$

(b) For $\frac{\pi}{2} \le \theta \le \pi$, there is one point *P* on the polar curve *r* with *x*-coordinate -3. Find the angle θ that corresponds to point *P*. Find the *y*-coordinate of point *P*. Show the work that leads to your answers.

$$\boldsymbol{\theta = 2.0169, \quad y = 6.2724}$$

(2) An object moving along a curve in the *xy*-plane has position $(x(t), y(t))$ at time $t \ge 0$ with $\frac{dx}{dt} = e^{-3t} + 7t$. The derivative of $\frac{dy}{dt}$ is not explicitly given. At time $t = 4$, the object is at position (3,4).

(a) Find the *x*-coordinate at $t = 1$. $\boldsymbol{x(1) = -49.5166}$

(b) For $t \ge 4$, the line tangent to the curve $(x(t), y(t))$ has a slope of $t^2 - 1$. Find the acceleration vector of the object a $t = 2$.

$$\boldsymbol{\langle 6.9926, 76.9876 \rangle}$$

(3) A particle is moving along a curve so that its position at time *t* is $(x(t), y(t))$, where $x(t) = t^2 - 4t + 8$ and $y(t)$ is not explicitly given. Both *x* and *y* are measured in meters, and *t* is measured in seconds. It is known that $\frac{dy}{dt} = te^{t-3} - 1$.

(a) Find the speed of the particle at time $t = 3$ seconds. $\boldsymbol{\sqrt{8} \text{ or } 2.8284}$

(b) Find the time t, $0 \le t \le 4$, when the line tangent to the path of the particle is horizontal. Is the direction of the motion of the particle toward the left or toward the right at that time? Give a reason for your answer.

$\boldsymbol{t = 2.2079 \;\; x'(2.2079) = .4158}$**. Since** $\boldsymbol{\frac{dx}{dt} > 0}$**, the particle is moving to the right.**

(c) There is point with x-coordinate 5 through which the particle passes twice. Determine the slopes of the lines tangent to the particle's path at that point.

$\boldsymbol{t = 1, t = 3 \; \frac{dy}{dx}(1) = 0.4323, \frac{dy}{dx}(3) = 1}$

(d) Using the conditions and solutions in part (c), find the y-coordinate of the point, given $y(2) = 3 + \frac{1}{e}$.

$$\boldsymbol{y = 4}$$

(4) A curve is described by the equation in polar coordinates $r = \theta + \sin(2\theta)$ for $0 \le \theta \le \pi$, where r is measured in meters and θ is measured in radians. The derivative of r with respect to θ is given by $\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$.

(a) For $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$, $\frac{dr}{d\theta}$ is negative. What does this fact say about r? What does this fact say about the curve?

Since $\boldsymbol{\frac{dr}{d\theta} < 0}$**, the radius is decreasing as the angle increases. Since** $\boldsymbol{r > 0}$**, the curve is moving towards the pole on the given interval.**

(b) Find the value of θ in the interval $0 \le \theta \le \frac{\pi}{2}$ that corresponds to the point on the curve in the first quadrant with greatest distance from the origin. Justify your answer.

$\boldsymbol{r = 1.9132 \; at \; \theta = 1.9132 \left(\frac{\pi}{3}\right) \;\; check \; endpoints \; to \; verify \; answers \; (0, 0) \; and \; \left(\frac{\pi}{2}, 1.5708\right)}$

(5) Find the common interior area shared by the curves $r = -5\cos\theta$ and $r = 2 - 2\cos\theta$. $\boldsymbol{14.9134}$

(6) The figure to the right shows the graphs of the polar curves $r = 2\cos(3\theta)$ and $r = 2$. What is the sum of the areas of the shaded regions?

(A) 0.858 (B) 3.142 (C) 8.566 **(D) 9.425** (E) 15.708