AP Calculus BC
Midterm Review – Taylor & Polar Series

14. The sum of the infinite geometric series $\frac{3}{2}+\frac{9}{16}+\frac{27}{128}+\frac{81}{1{,}024}+\ldots$ is

(A) 1.60 (B) 2.35 (C) 2.40 (D) 2.45 (E) 2.50

17. Let f be the function given by $f(x)=\ln(3-x)$. The third-degree Taylor polynomial for f about $x=2$ is

(A) $-(x-2)+\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$

(B) $-(x-2)-\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$

(C) $(x-2)+(x-2)^2+(x-2)^3$

(D) $(x-2)+\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$

(E) $(x-2)-\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$

20. What are all values of x for which the series $\sum_{n=1}^{\infty}\frac{(x-2)^n}{n\cdot 3^n}$ converges?

(A) $-3\le x\le 3$
(B) $-3<x<3$
(C) $-1<x\le 5$
(D) $-1\le x\le 5$
(E) $-1\le x<5$

24. The Taylor series for $\sin x$ about $x=0$ is $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots$. If f is a function such that $f'(x)=\sin\left(x^2\right)$, then the coefficient of x^7 in the Taylor series for $f(x)$ about $x=0$ is

(A) $\frac{1}{7!}$ (B) $\frac{1}{7}$ (C) 0 (D) $-\frac{1}{42}$ (E) $-\frac{1}{7!}$

76. For what integer k, $k>1$, will both $\sum_{n=1}^{\infty}\frac{(-1)^{kn}}{n}$ and $\sum_{n=1}^{\infty}\left(\frac{k}{4}\right)^n$ converge?

(A) 6 (B) 5 (C) 4 (D) 3 (E) 2

76. Which of the following sequences converge?

I. $\left\{\frac{5n}{2n-1}\right\}$

II. $\left\{\frac{e^n}{n}\right\}$

III. $\left\{\frac{e^n}{1+e^n}\right\}$

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

18. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n}{n+2}$ II. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$ III. $\sum_{n=1}^{\infty} \frac{1}{n}$

(A) None
(B) II only
(C) III only
(D) I and II only
(E) I and III only

22. If $\lim_{b\to\infty} \int_1^b \frac{dx}{x^p}$ is finite, then which of the following must be true?

(A) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges

(B) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges

(C) $\sum_{n=1}^{\infty} \frac{1}{n^{p-2}}$ converges

(D) $\sum_{n=1}^{\infty} \frac{1}{n^{p-1}}$ converges

(E) $\sum_{n=1}^{\infty} \frac{1}{n^{p+1}}$ diverges

84. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{\sqrt{n}}$ converges?

(A) $-3 < x < -1$ (B) $-3 \le x < -1$ (C) $-3 \le x \le -1$ (D) $-1 \le x < 1$ (E) $-1 \le x \le 1$

6) What are all values of p for which $\int_1^{\infty} \frac{1}{x^{2p}} dx$ converges?

(A) $p < -1$
(B) $p > 0$
(C) $p > \frac{1}{2}$
(D) $p > 1$
(E) There are no values of p for which this integral converges.

10) What is the value of $\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n}$?

(A) 1
(B) 2
(C) 4
(D) 6
(E) The series diverges.

22) What are all values of p for which the infinite series $\sum_{n=1}^{\infty} \frac{n}{n^p + 1}$ converges?

(A) $p > 0$
(B) $p \geq 1$
(C) $p > 1$
(D) $p \geq 2$
(E) $p > 2$

FRQS:

1. Calculator Inactive

The function g has derivatives of all orders, and the Maclaurin series for g is
$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+3} = \frac{x}{3} - \frac{x^3}{5} + \frac{x^5}{7} - \cdots$.

(a) Using the ratio test, determine the interval of convergence of the Maclaurin series for g.

(b) The Maclaurin series for g evaluated at $x = \frac{1}{2}$ is an alternating series whose terms decrease in absolute value to 0. The approximation for $g\left(\frac{1}{2}\right)$ using the first two nonzero terms of this series is $\frac{17}{120}$. Show that this approximation differs from $g\left(\frac{1}{2}\right)$ by less than $\frac{1}{200}$.

(c) Write the first three nonzero terms and the general term of the Maclaurin series for $g'(x)$.

2. Calculator Inactive

A function f has derivatives of all orders at $x = 0$. Let $P_n(x)$ denote the nth-degree Taylor polynomial for f about $x = 0$.

(a) It is known that $f(0) = -4$ and that $P_1\left(\frac{1}{2}\right) = -3$. Show that $f'(0) = 2$.

(b) It is known that $f''(0) = -\frac{2}{3}$ and $f'''(0) = \frac{1}{3}$. Find $P_3(x)$.

(c) The function h has first derivative given by $h'(x) = f(2x)$. It is known that $h(0) = 7$. Find the third-degree Taylor polynomial for h about $x = 0$.