AP Calculus BC
Midterm Review – Advanced Integration

11. $\int_{1}^{\infty} \frac{x}{(1+x^2)^2}\,dx$ is

(A) $-\frac{1}{2}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) divergent

84. $\int x^2 \sin x\,dx =$

(A) $-x^2\cos x - 2x\sin x - 2\cos x + C$

(B) $-x^2\cos x + 2x\sin x - 2\cos x + C$

(C) $-x^2\cos x + 2x\sin x + 2\cos x + C$

(D) $-\frac{x^3}{3}\cos x + C$

(E) $2x\cos x + C$

86. $\int \frac{dx}{(x-1)(x+3)} =$

(A) $\frac{1}{4}\ln\left|\frac{x-1}{x+3}\right| + C$

(B) $\frac{1}{4}\ln\left|\frac{x+3}{x-1}\right| + C$

(C) $\frac{1}{2}\ln\left|(x-1)(x+3)\right| + C$

(D) $\frac{1}{2}\ln\left|\frac{2x+2}{(x-1)(x+3)}\right| + C$

(E) $\ln\left|(x-1)(x+3)\right| + C$

4. $\int \frac{1}{x^2-6x+8}\,dx =$

(A) $\frac{1}{2}\ln\left|\frac{x-4}{x-2}\right| + C$

(B) $\frac{1}{2}\ln\left|\frac{x-2}{x-4}\right| + C$

(C) $\frac{1}{2}\ln\left|(x-2)(x-4)\right| + C$

(D) $\frac{1}{2}\ln\left|(x-4)(x+2)\right| + C$

(E) $\ln\left|(x-2)(x-4)\right| + C$

15. $\int x\cos x\,dx =$

(A) $x\sin x - \cos x + C$

(B) $x\sin x + \cos x + C$

(C) $-x\sin x + \cos x + C$

(D) $x\sin x + C$

(E) $\frac{1}{2}x^2\sin x + C$

25. $\int_0^{\infty} x^2 e^{-x^3}\, dx$ is

(A) $-\frac{1}{3}$ (B) 0 (C) $\frac{1}{3}$ (D) 1 (E) divergent

26. The population $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt} = P\left(2 - \frac{P}{5000}\right)$, where the initial population $P(0) = 3,000$ and t is the time in years. What is $\lim_{t\to\infty} P(t)$?

(A) $2,500$ (B) $3,000$ (C) $4,200$ (D) $5,000$ (E) $10,000$

28. $\lim_{x\to 1} \frac{\int_1^x e^{t^2}\, dt}{x^2 - 1}$ is

(A) 0 (B) 1 (C) $\frac{e}{2}$ (D) e (E) nonexistent

Short Answer:

1. $\int_0^{\pi/2} \tan\theta \, d\theta$

2. The populator of $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt} = P\left(2 - \frac{P}{5000}\right)$, where the initial population is $P(0) = 3000$ and t is the time in years. What is $\lim_{t\to\infty} P(t)$?

3. $\int_{-1}^{\infty} \frac{1}{x^2+5x+6}\, dx$

4. The function N satisfies the logistic differential equation $\frac{dN}{dt} = \frac{N}{10}\left(1 - \frac{N}{850}\right)$, where $N(0)$=105. Which of the following statements is false?

(A) $\lim_{t\to\infty} N(t) = 850$
(B) $\frac{dN}{dt}$ has a maximum value when $N = 105$
(C) $\frac{d^2N}{dt^2} = 0$ when $N = 425$.
(D) When $N > 425$, $\frac{dN}{dt} > 0$ and $\frac{d^2N}{dt^2} < 0$.