## AP Calculus - Basic Integration Practice Problems - Calculator Inactive

Name: $\qquad$
(1) $\int_{0}^{1} x \sqrt{a x^{2}+b} d x$, where $a$ and $b$ are constants.

For Questions 2-4, the graph of a function $f$ consists of a semicircle and two line segments as shown to the right. Let $g(x)=\int_{1}^{x} f(t) d t$.
(2) Find $g(3)$.
(3) Find $g(-1)$.

(4) Write an equation for the line tangent to the graph of $g$ at $x=-1$.
(5) The rate of water flow, in gallons per hour, can be modeled by $R(t)=\frac{1}{2}\left(10+6 t-\frac{1}{4} t^{2}\right)$. Find the average water flow over the first 6 hours. Indicate units of measure.
(6) Using the substitution $u=\sqrt{x}, \int_{1}^{4} \frac{e^{\sqrt{x}}}{\sqrt{x}} d x$ is equal to which of the following?
(A) $2 \int_{1}^{16} e^{u} d u$
(B) $2 \int_{1}^{4} e^{u} d u$
(C) $2 \int_{1}^{2} e^{u} d u$
(D) $\frac{1}{2} \int_{1}^{2} e^{u} d u$
(E) $\int_{1}^{4} e^{u} d u$
(7) $\frac{d}{d x}\left(\int_{x^{4}}^{\sin 2 x} \sqrt{x} d x\right)=$
(8) The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function $R$ of time $t$. The table below shows the rate as measured every 3 hours for a 24 -hour period.

| $t$ (hours) | 0 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $R(t)(\mathrm{gal} / \mathrm{hr})$ | 2 | 3 | 5 | 6 | 6 | 8 | 10 | 11 | 14 |

Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate the value of $\int_{0}^{24} R(t) d t$. Using correct units, explain the meaning of your answer in terms of water flow.
(9) $\int_{\pi / 6}^{\pi / 2} \cot x d x=$
(A) $\ln \frac{1}{2}$
(B) $\frac{1}{2}$
(C) $-\ln \frac{1}{2}$
(D) $\ln (\sqrt{3}-1)$
(E) None of these
(10) If $f^{\prime}(x)$ is positive and $f^{\prime}(x)$ is negative, which approximation is an underestimation?
I. Left RAM
II. Right RAM
III. Trapezoidal Rule
I only
B. II only
C. III. only
D. I and III
E. II and III
(11) $\int_{0}^{1} \frac{e^{x}}{\left(3-e^{x}\right)^{2}} d x=$
(A) $3 \ln |e-3|$
(B) 1
(C) $\frac{1}{3-e}$
(D) $\frac{e-1}{2(3-e)}$
(E) $\frac{e-2}{3-e}$
(12) The acceleration of a particle moving along a straight line is given by $a=6 t$. If, when $t=0$ its velocity is $v=1$ and its distance $s=3$, then determine the position function for any time $t$.
(13) $\int \frac{2 y+3}{\sqrt{y-5}} d y=$
(14) If $\int_{a}^{b} f(x) d x=3$ and $\int_{a}^{b} g(x) d x=-2$, then which of the following must be true?
I. $f(x)>g(x)$ for all $a \leq x \leq b$
II. $\int_{a}^{b}[f(x)+g(x)] d x=1$
III. $\int_{a}^{b}[f(x) g(x)] d x=-6$
(A) I only
(B) II only
(C) III only
(D) II and III only
(E) I, II, and III
(15) $\int_{-2}^{3}|x+1| d x=$

