MIDTERM SET: 3

1. $\lim\limits_{x\to\infty}\dfrac{(2x-1)(3-x)}{(x-1)(x+3)}$ is $\dfrac{-2x^2}{x^2}$

(A) -3 $\quad$ $\boxed{\text{(B) } -2}$ $\quad$ (C) 2 $\quad$ (D) 3 $\quad$ (E) nonexistent

3. If $f(x)=(x-1)(x^2+2)^3$, then $f'(x)=$

(A) $6x(x^2+2)^2$

(B) $6x(x-1)(x^2+2)^2$

(C) $(x^2+2)^2(x^2+3x-1)$

$\boxed{\text{(D) } (x^2+2)^2(7x^2-6x+2)}$

(E) $-3(x-1)(x^2+2)^2$

$f'(x)=(x-1)\left[3(x^2+3)^2(2x)\right]+(x^2+2)^3(1)$

$f'(x)=(x^2+3)^2\left[6x(x-1)\right]+(x^2+2)$

$f'(x)=(x^2+3)^2(6x^2-6x+x^2+2)$

$f'(x)=(x^2+3)^2(7x^2-6x+2)$

5. $\lim\limits_{x\to 0}\dfrac{5x^4+8x^2}{3x^4-16x^2}$ is $\lim\limits_{x\to 0}\dfrac{\cancel{x^2}(5x^2+8)}{\cancel{x^2}(3x^2-16)}=\dfrac{8}{-16}$

$\boxed{\text{(A) } -\frac{1}{2}}$ $\quad$ (B) 0 $\quad$ (C) 1 $\quad$ (D) $\frac{5}{3}+1$ $\quad$ (E) nonexistent

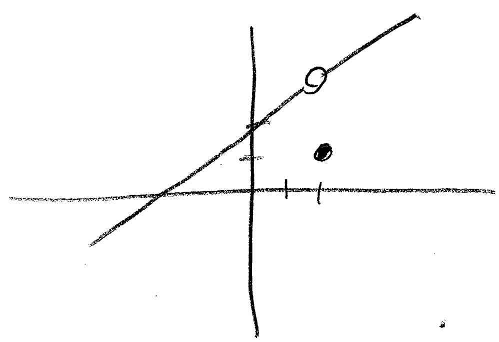
$$f(x)=\begin{cases}\dfrac{x^2-4}{x-2} & \text{if } x\neq 2\\ 1 & \text{if } x=2\end{cases} \rightarrow \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}}=x+2$$

6. Let f be the function defined above. Which of the following statements about f are true?

I. f has a limit at $x=2$. ✓

II. f is continuous at $x=2$. ✗

III. f is differentiable at $x=2$. ✗



(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

A particle moves along the x-axis with velocity given by $v(t)=3t^2+6t$ for time $t \geq 0$. If the particle is at position $x=2$ at time $t=0$, what is the position of the particle at $t=1$?

(A) 4 $\qquad$ $\boxed{\text{(B) 6}}$ $\qquad$ (C) 9 $\qquad$ (D) 11 $\qquad$ (E) 12

$s=\int(3t^2+6t)\,dt$

$s=t^3+3t^2+C$

$2=0+0+c$

$s=t^3+3t^2+2$

$s(1)=1+3+2$

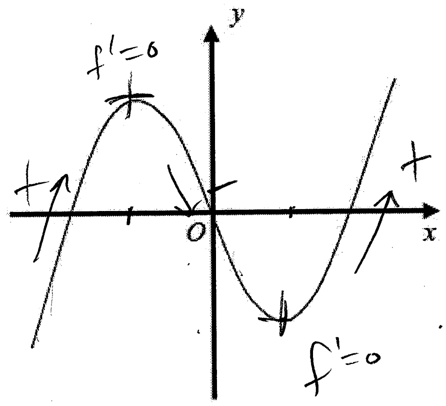
8. If $f(x)=\cos(3x)$, then $f'\left(\dfrac{\pi}{9}\right)=$

(A) $\dfrac{3\sqrt{3}}{2}$ $\qquad$ (B) $\dfrac{\sqrt{3}}{2}$ $\qquad$ (C) $-\dfrac{\sqrt{3}}{2}$ $\qquad$ (D) $-\dfrac{3}{2}$ $\qquad$ (E) $-\dfrac{3\sqrt{3}}{2}$

$f'(x)=-3\sin(3x)$

$f'\left(\frac{\pi}{9}\right)=-3\sin\left(\frac{\pi}{3}\right)$

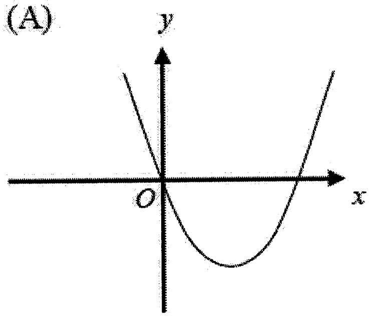
$=-3\left(\frac{\sqrt{3}}{2}\right)$



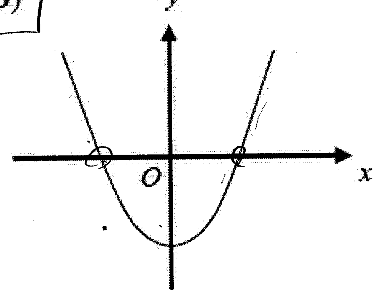
Graph of f

11. The graph of a function f is shown above. Which of the following could be the graph of f', the derivative of f?

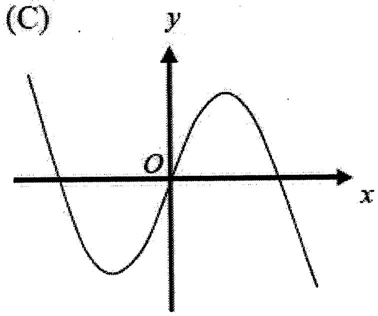
(A)



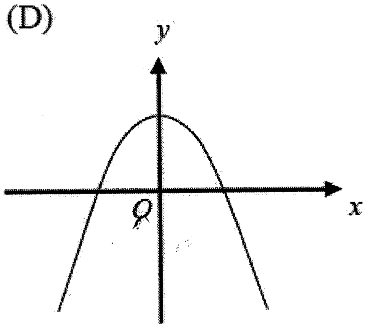
(B)



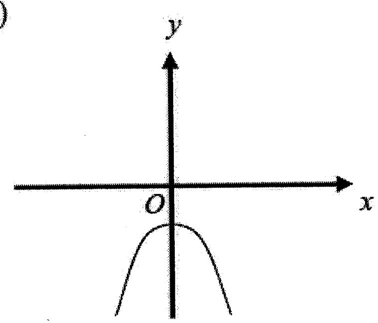
(C)



(D)



(E)



If $f(x)=e^{(2/x)}$, then $f'(x)=$ $f'(x)=e^{(2/x)}\cdot(-2x^{-2}) = \frac{-2e^{(2/x)}}{x^2}$

$2x^{-1}$

(A) $2e^{(2/x)}\ln x$ (B) $e^{(2/x)}$ (C) $e^{(-2/x^2)}$ (D) $\boxed{-\frac{2}{x^2}e^{(2/x)}}$ (E) $-2x^2e^{(2/x)}$

13. If $f(x)=x^2+2x$, then $\frac{d}{dx}(f(\ln x))=$ $\frac{d}{dx}\left[(\ln x)^2+2\ln x\right] = 2\ln x\left(\frac{1}{x}\right)+\frac{2}{x} = \frac{2\ln x}{x}+\frac{2}{x}$

(A) $\frac{2\ln x+2}{x}$ (B) $2x\ln x+2$ (C) $2\ln x+2$ (D) $2\ln x+\frac{2}{x}$ (E) $\frac{2x+2}{x}$

x	0	1	2	3
$f''(x)$	5	0	−7	4

14. The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?

(A) f is increasing on the interval $(0, 2)$.

(B) f is decreasing on the interval $(0, 2)$.

(C) f has a local maximum at $x=1$.

(D) The graph of f has a point of inflection at $x=1$.

(E) The graph of f changes concavity in the interval $(0, 2)$.

16. If $\sin(xy)=x$, then $\frac{dy}{dx}=$

$\cos(xy)\cdot\left(x\,\frac{dy}{dx}+y\right)=1$

$x\frac{dy}{dx}+y = \frac{1}{\cos(xy)}$

$x\frac{dy}{dx} = \frac{1}{\cos(xy)} - y$

$x\frac{dy}{dx} = \frac{1-y\cos(xy)}{\cos(xy)}$

$dy/dx = \frac{1-y\cos(xy)}{x\cos(xy)}$

(A) $\frac{1}{\cos(xy)}$

(B) $\frac{1}{x\cos(xy)}$

(C) $\frac{1-\cos(xy)}{\cos(xy)}$

(D) $\boxed{\frac{1-y\cos(xy)}{x\cos(xy)}}$

(E) $\frac{y(1-\cos(xy))}{x}$

$m = -1$

In the xy-plane, the line $x + y = k$, where k is a constant, is tangent to the graph of $y = x^2 + 3x + 1$. What is the value of k?

$y' = 2x + 3$

$-1 = 2x + 3$

$2x = -4$

$x = -2$

$y = 4 - 6 + 1 = -1$

$\boxed{\text{(A) } -3}$ (B) -2 (C) -1 (D) 0 (E) 1

19. What are all horizontal asymptotes of the graph of $y = \dfrac{5 + 2^x}{1 - 2^x}$ in the xy-plane?

(A) $y = -1$ only

(B) $y = 0$ only

(C) $y = 5$ only

(D) $y = -1$ and $y = 0$

$\boxed{\text{(E) } y = -1 \text{ and } y = 5}$

$\lim_{x \to \infty} \dfrac{5 + 2^x}{1 - 2^x} \cdot \dfrac{\frac{1}{2^x}}{\frac{1}{2^x}}$

$\lim_{x \to \infty} \dfrac{\frac{5}{2^x} + 1}{\frac{1}{2^x} - 1} = \dfrac{0 + 1}{0 - 1} = \boxed{-1}$

$\lim_{x \to -\infty} \dfrac{5 + 2^x}{1 - 2^x} = \dfrac{5 + \frac{1}{2^\infty}}{1 - \frac{1}{2^\infty}} = \dfrac{5 + 0}{1 - 0} = \boxed{5}$

* CONSIDER $x \to \infty / x \to -\infty$

20. Let f be a function with a second derivative given by $f''(x) = x^2(x-3)(x-6)$. What are the x-coordinates of the points of inflection of the graph of f?

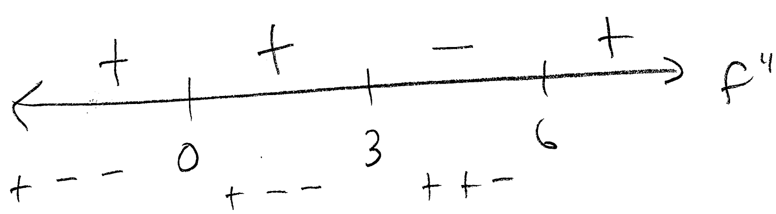
(A) 0 only

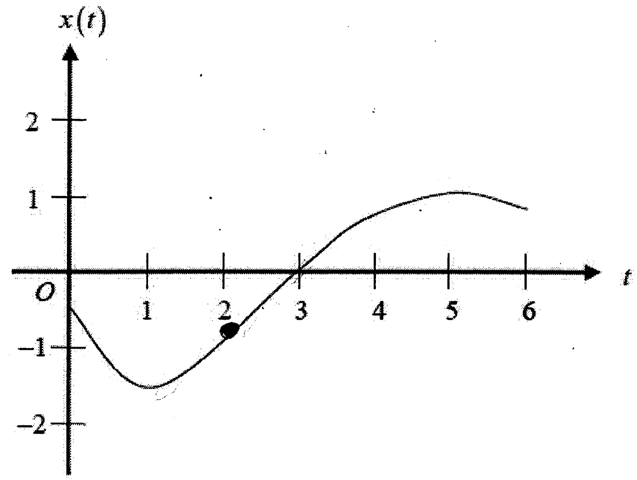
(B) 3 only

(C) 0 and 6 only

$\boxed{\text{(D) } 3 \text{ and } 6 \text{ only}}$

(E) 0, 3, and 6





21. A particle moves along a straight line. The graph of the particle's position $x(t)$ at time t is shown above for $0 < t < 6$. The graph has horizontal tangents at $t = 1$ and $t = 5$ and a point of inflection at $t = 2$. For what values of t is the velocity of the particle increasing?

(A) $0 < t < 2$

$\Rightarrow f'' > 0$

CONCAVE UP

(B) $1 < t < 5$

(C) $2 < t < 6$

(D) $3 < t < 5$ only

(E) $1 < t < 2$ and $5 < t < 6$

$y - 1 = 4(x-2)$

$y = 1 + 4(x-2)$

24. The function f is twice differentiable with $f(2) = 1$, $f'(2) = 4$, and $f''(2) = 3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of f at $x = 2$?

(A) 0.4 (B) 0.6 (C) 0.7 (D) 1.3 (E) 1.4

$f(1.9) \approx 1 + 4(1.9 - 2)$

$= 1 + 4(-.1)$

$$f(x) = \begin{cases} cx + d & \text{for } x \le 2 \\ x^2 - cx & \text{for } x > 2 \end{cases}$$

25. Let f be the function defined above, where c and d are constants. If f is differentiable at $x = 2$, what is the value of $c + d$?

@ $x = 2$

$f'(x) \Rightarrow c = 2x - c$

(A) -4 (B) -2 (C) 0 (D) 2 (E) 4

$c = 4 - c$

$2c = 4$

$c = 2$

26. What is the slope of the line tangent to the curve $y = \arctan(4x)$ at the point at which $x = \frac{1}{4}$?

$y' = \frac{4}{1 + 16x^2} = \frac{4}{1 + 16(1/16)} = \frac{4}{2}$

CONT: $cx + d = x^2 - cx$

(A) 2 (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) -2

$2(2) + d = 4 - 4$

$d = -4$

Let f be a differentiable function such that $f(3)=15$, $f(6)=3$, $f'(3)=-8$, and $f'(6)=-2$. The function g is differentiable and $g(x)=f^{-1}(x)$ for all x. What is the value of $g'(3)$? $(3, \frac{6}{\ })$

$g'(3) = \frac{1}{f'(6)} = \frac{1}{-2}$

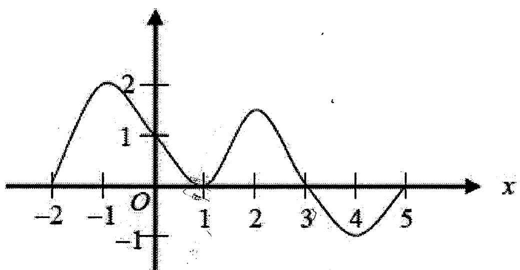
$\boxed{\text{(A) } -\frac{1}{2}}$

(B) $-\frac{1}{8}$

(C) $\frac{1}{6}$

(D) $\frac{1}{3}$

(E) The value of $g'(3)$ cannot be determined from the information given.



Graph of f'

76. The graph of f', the derivative f, is shown above for $-2 \le x \le 5$. On what intervals is f increasing?

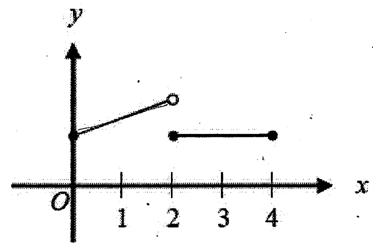
(A) [−2, 1] only

$\boxed{\text{(B) } [-2, 3]}$ $[-2,1] \cup [1,3]$

(C) [3, 5] only

(D) [0, 1.5] and [3, 5]

(E) [−2, −1], [1, 2], and [4, 5]



Graph of f

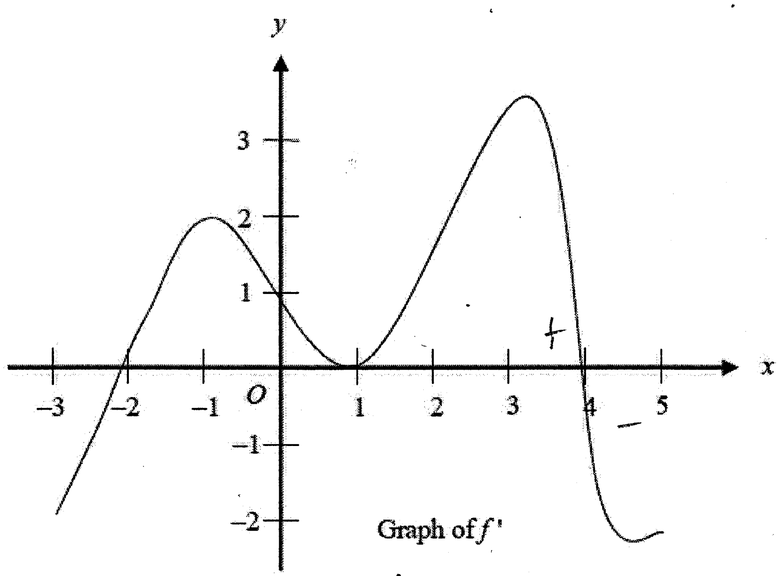
77. The figure above shows the graph of a function f with domain $0 \le x \le 4$. Which of the following statements are true?

I. $\lim_{x \to 2^-} f(x)$ exists. ✓

II. $\lim_{x \to 2^+} f(x)$ exists. ✓

III. $\lim_{x \to 2} f(x)$ exists. X

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III



Graph of f'

84. The graph of the derivative of a function f is shown in the figure above. The graph has horizontal tangent lines at $x=-1$, $x=1$, and $x=3$. At which of the following values of x does f have a relative maximum? $+/-$

(A) -2 only

(B) 1 only

(C) 4 only

(D) -1 and 3 only

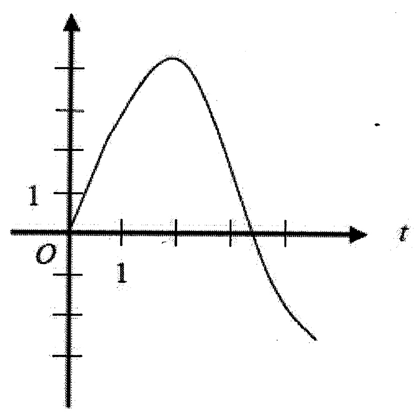
(E) -2, 1, and 4

t	0	1	2	3	4
$v(t)$	-1	2	3	0	-4

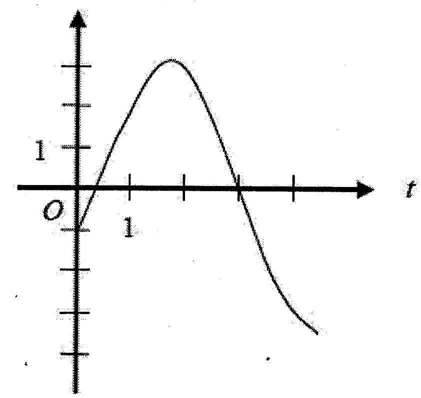
↑ Slope 2 @ x=1 ↑ Horz TAN

86. The table gives selected values of the velocity, $v(t)$, of a particle moving along the x-axis. At time $t=0$, the particle is at the origin. Which of the following could be the graph of the position, $x(t)$, of the particle for $0 \le t \le 4$?

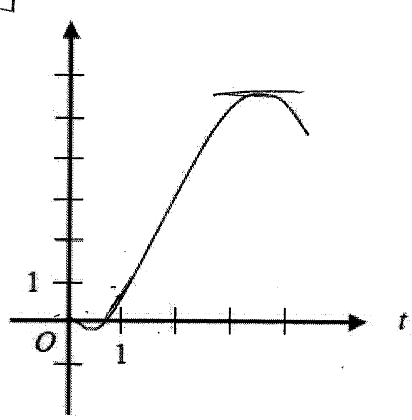
(A) $x(t)$



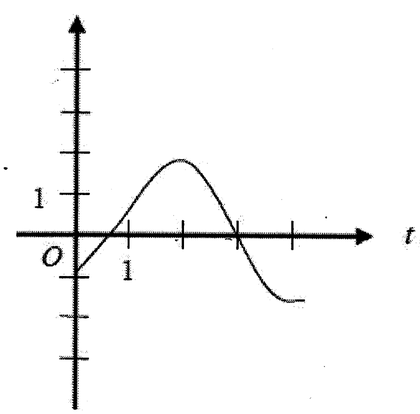
(B) $x(t)$



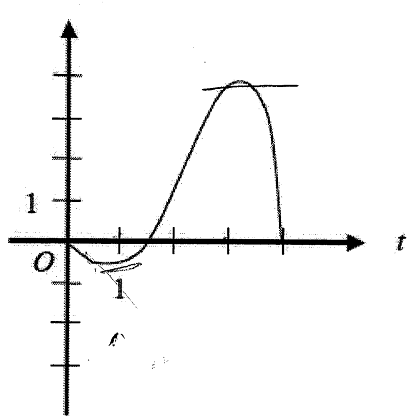
(C) $x(t)$



(D) $x(t)$



(E) $x(t)$



The radius of a sphere is decreasing at a rate of 2 centimeters per second. At the instant when the radius of the sphere is 3 centimeters, what is the rate of change, in square centimeters per second, of the surface area of the sphere? (The surface area S of a sphere with radius r is $S = 4\pi r^2$)

$S = 4\pi r^2$

(A) -108π (B) -72π $\boxed{\text{(C) } -48\pi}$ (D) -24π (E) -16π

$dS/dt = 8\pi r \, dr/dt$

89. The function f is continuous for $-2 \le x \le 2$ and $f(-2) = f(2) = 0$. If there is no c, where $-2 < c < 2$, for which $f'(c) = 0$, which of the following statements must be true?

Rolles Thm

$\frac{dS}{dt} = 8\pi(3)(-2)$

(A) For $-2 < k < 2$, $f'(k) > 0$.

(B) For $-2 < k < 2$, $f'(k) < 0$.

(C) For $-2 < k < 2$, $f'(k)$ exists.

(D) For $-2 < k < 2$, $f'(k)$ exists, but f' is not continuous.

$\boxed{\text{(E) For some } k, \text{ where } -2 < k < 2, \ f'(k) \text{ does not exist.}}$

90. The function f is continuous on the closed interval [2, 4] and twice differentiable on the open interval (2, 4). If $f'(3) = 2$ and $f''(x) < 0$ on the open interval (2,4), which of the following could be a table of values for f?

over

(A)

x	$f(x)$
2	2.5
3	5
4	6.5

(B)

x	$f(x)$
2	2.5
3	5
4	7

(C)

x	$f(x)$
2	~~3~~
3	5
4	6.5

$y - 5 = 2(x-3)$

$y = 5 + 2(x-3)$

$y(2) \approx 5 + 2(-1) = 3$

$y(4) \approx 5 + 2(1) = 7$

(D)

x	$f(x)$
2	~~3~~
3	5
4	7

(E)

x	$f(x)$
2	~~3.5~~
3	5
4	7.5