2. $\lim_{x \to 0} \frac{2x^6 + 6x^3}{4x^5 + 3x^3}$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 $\boxed{\text{(D) 2}}$ (E) nonexistent

$\overset{LH}{=} \frac{12x^5 + 18x^2}{20x^4 + 9x^2} \overset{LH}{=} \frac{60x^4 + 36x}{80x^3 + 18x} \overset{LH}{=} \frac{\cancel{240x^3} + 36}{\cancel{240x^2} + 18}$

or $\lim_{x \to 0} \frac{2\cancel{x^3}(x^3 + 3)}{\cancel{x^3}(4x^2 + 3)}$

$\lim_{x \to 0} \frac{2(x^3+3)}{4x^2+3} = \frac{6}{3}$

$$f(x) = \begin{cases} x^2 - 3x + 9 & \text{for } x \le 2 \\ kx + 1 & \text{for } x > 2 \end{cases}$$

3. The function f is defined above. For what value of k, if any, is f continuous at $x = 2$?

(A) 1

(B) 2

$\boxed{\text{(C) 3}}$

(D) 7

(E) No value of k will make f continuous at $x = 2$.

$4 - 6 + 9 = 2k + 1$

$7 = 2k + 1$

$6 = 2k$

4. If $f(x) = \cos^3(4x)$, then $f'(x) =$

(A) $3\cos^2(4x)$

$\boxed{\text{(B) } -12\cos^2(4x)\sin(4x)}$

(C) $-3\cos^2(4x)\sin(4x)$

(D) $12\cos^2(4x)\sin(4x)$

(E) $-4\sin^3(4x)$

$f(x) = (\cos(4x))^3$

$f'(x) = \underbrace{3(\cos 4(x))^2}_{E} \underbrace{(-\sin(4x)}_{I} \cdot \underset{A}{4}$

$f'(x) = -12\cos^2 4x \sin 4x$

5. The function f given by $f(x) = 2x^3 - 3x^2 - 12x$ has a relative minimum at $x =$

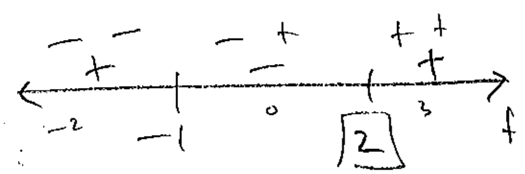
(A) -1 (B) 0 $\boxed{\text{(C) 2}}$ (D) $\frac{3 - \sqrt{105}}{4}$ (E) $\frac{3 + \sqrt{105}}{4}$

$f'(x) = 6x^2 - 6x - 12$

$6(x^2 - x - 2) = 0$

$(x - 2)(x + 1) = 0$

$x = 2 \quad x = -1$



6. Let f be the function given by $f(x) = (2x - 1)^5(x + 1)$. Which of the following is an equation for the line tangent to the graph of f at the point where $x = 1$?

(A) $y = 21x + 2$

$\boxed{\text{(B) } y = 21x - 19}$

(C) $y = 11x - 9$

(D) $y = 10x + 2$

(E) $y = 10x - 8$

$f(1) = (2-1)^5(1+1)$

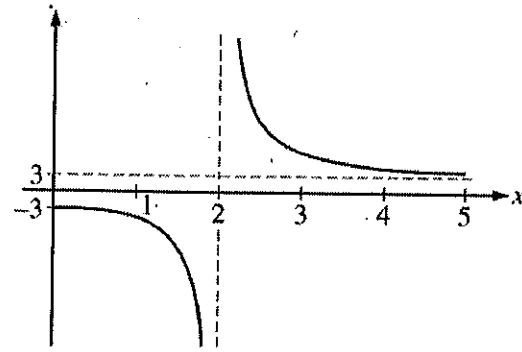
$f(1) = 1 \cdot 2 = 2$

$f'(x) = (2x-1)^5(1) + (x+1)[5(2x-1)^4(2)]$

$f'(1) = 1 + 2[10] = 21$

$y - 2 = 21(x - 1)$

$\boxed{y = 21x - 19}$



← HA $y=3$

$a=3$

VA $x=2$ $\quad f(x)=\dfrac{3a^2+12}{(x+2)(x-2)}$ $\quad b=-4$

10. The function f is given by $f(x)=\dfrac{ax^2+12}{x^2+b}$. The figure above shows a portion of the graph of f. Which of the following could be the values of the constants a and b?

(A) $a=-3,\ b=2$

(B) $a=2,\ b=-3$

(C) $a=2,\ b=-2$

[✓ (D) $a=3,\ b=-4$]

(E) $a=3,\ b=4$

$$y'=\frac{(x+1)(-e^{-x})-(e^{-x})(1)}{(x+1)^2}$$

11. What is the slope of the line tangent to the graph of $y=\dfrac{e^{-x}}{x+1}$ at $x=1$?

$$y'(1)=\frac{(2)(-1/e)-(1/e)}{4}$$

(A) $-\dfrac{1}{e}$ [✓ (B) $-\dfrac{3}{4e}$] (C) $-\dfrac{1}{4e}$ (D) $\dfrac{1}{4e}$ (E) $\dfrac{1}{e}$

$$y'(1)=\frac{-3/e}{4}$$

14. $\displaystyle\lim_{h\to 0}\frac{e^{(2+h)}-e^2}{h}=$

(A) 0 (B) 1 (C) $2e$ [✓ (D) e^2] (E) $2e^2$

$$\frac{d}{dx}[e^x]=e^x\Big|_{x=2}$$

19. Let f be the function given by $f(x)=x^3-6x^2$. The graph of f is concave up when

[✓ (A) $x>2$]

(B) $x<2$

(C) $0<x<4$

(D) $x<0$ or $x>4$ only

(E) $x>6$ only

$f'(x)=3x^2-12x$

$f''(x)=6x-12$

$x=2$

$-$ $+$ f''
0 2 3

$(2,\infty)$ CONCAVE UP.

22. If $f'(x)=(x-2)(x-3)^2(x-4)^3$, then f has which of the following relative extrema?

I. A relative maximum at $x=2$ ✓

~~II. A relative minimum at x~~ $=3$

III. A relative ~~maximum~~ at $x=4$

[✓ (A) I only]

(B) III only

(C) I and III only

(D) II and III only

(E) I, II, and III

$(-\ +\ -)$ $+\ +\ -$ $+\ +\ -$ $+\ +\ +$

$+$ $-$ $-$ $+$ f'

0 2 2.5 3 3.5 4 5

$x=2$ REL MAX $\quad x=3$ NONE

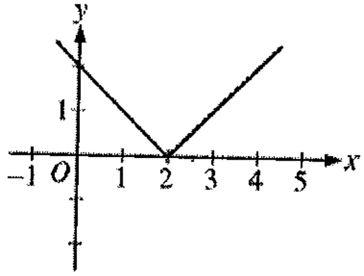
$x=4$ REL MIN

ALWAYS + SO f IS ALWAYS INCREASING (+SLOPE)

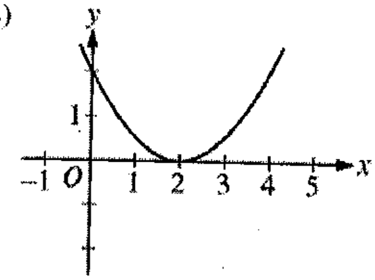
16. If $f'(x) = |x - 2|$, which of the following could be the graph of $y = f(x)$?

$f' = 0$ @ $x = 2$

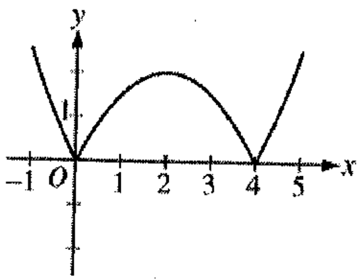
(A)



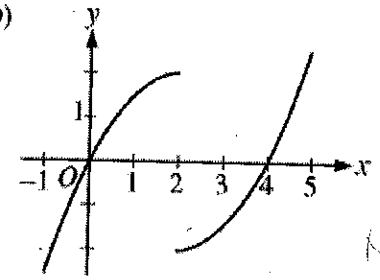
(B)



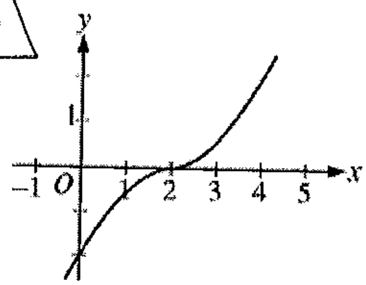
(C)

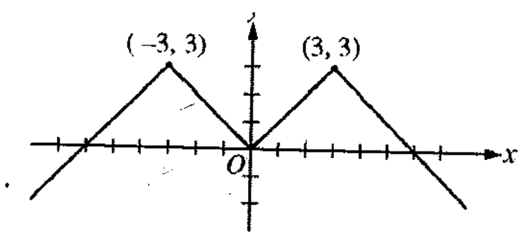


(D)



(E)





23. The graph of the even function $y = f(x)$ consists of 4 line segments, as shown above. Which of the following statements about f is false?

(A) $\lim_{x\to 0}\left(f(x) - f(0)\right) = 0$

* (B) $\lim_{x\to 0}\dfrac{f(x) - f(0)}{x} = 0$

(C) $\lim_{x\to 0}\dfrac{f(x) - f(-x)}{2x} = 0$

(D) $\lim_{x\to 2}\dfrac{f(x) - f(2)}{x - 2} = 1$

(E) $\lim_{x\to 3}\dfrac{f(x) - f(3)}{x - 3}$ does not exist.

25. If $x^2y - 3x = y^3 - 3$, then at the point $(-1, 2)$, $\frac{dy}{dx} =$ (Walk e END)

(A) $-\frac{7}{11}$ (B) $-\frac{7}{13}$ (C) $-\frac{1}{2}$ (D) $-\frac{3}{14}$ (E) 7

26. For $x > 0$, f is a function such that $f'(x) = \frac{\ln x}{x}$ and $f''(x) = \frac{1 - \ln x}{x^2}$. Which of the following is true?

(A) f is decreasing for $x > 1$, and the graph of f is concave down for $x > e$.

(B) f is decreasing for $x > 1$, and the graph of f is concave up for $x > e$. (@ END)

(C) f is increasing for $x > 1$, and the graph of f is concave down for $x > e$. ✓

(D) f is increasing for $x > 1$, and the graph of f is concave up for $x > e$.

(E) f is increasing for $0 < x < e$, and the graph of f is concave down for $0 < x < e^{3/2}$.

28. If $y = \sin^{-1}(5x)$, then $\frac{dy}{dx} =$

(A) $\frac{1}{1+25x^2}$

(B) $\frac{5}{1+25x^2}$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(5x)^2}} \cdot 5$$

(C) $\frac{-5}{\sqrt{1-25x^2}}$

(D) $\frac{1}{\sqrt{1-25x^2}}$

(E) $\frac{5}{\sqrt{1-25x^2}}$

$$\frac{dy}{dx} = \frac{5}{\sqrt{1-25x^2}}$$

78. For $t \geq 0$ hours, H is a differentiable function of t that gives the temperature, in degrees Celsius, at an Arctic weather station. Which of the following is the best interpretation of $H'(24)$?

(A) The change in temperature during the first day

(B) The change in temperature during the 24th hour

(C) The average rate at which the temperature changed during the 24th hour

(D) The rate at which the temperature is changing during the first day

(E) The rate at which the temperature is changing at the end of the 24th hour

82. If f is a continuous function on the closed interval $[a, b]$, which of the following must be true?

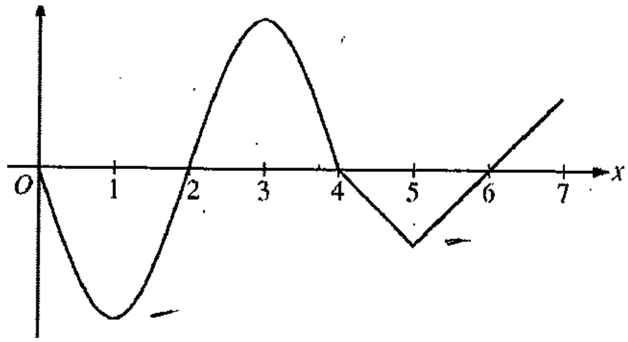
(A) There is a number c in the open interval (a, b) such that $f(c) = 0$.

(B) There is a number c in the open interval (a, b) such that $f(a) < f(c) < f(b)$.

(C) There is a number c in the closed interval $[a, b]$ such that $f(c) \geq f(x)$ for all x in $[a, b]$.

(D) There is a number c in the open interval (a, b) such that $f'(c) = 0$.

(E) There is a number c in the open interval (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.



Graph of f'

84. The graph of f', the derivative of the function f, is shown above. On which of the following intervals is f decreasing?

(A) $[2, 4]$ only

(B) $[3, 5]$ only

(C) $[0, 1]$ and $[3, 5]$

(D) $[2, 4]$ and $[6, 7]$

(E) $[0, 2]$ and $[4, 6]$

x	3	4	5	6	7
$f(x)$	20	17	12	16	20

86. The function f is continuous and differentiable on the closed interval $[3, 7]$. The table above gives selected values of f on this interval. Which of the following statements must be true?

~~I.~~ The minimum value of f on $[3, 7]$ is 12.

II. There exists c, for $3 < c < 7$, such that $f'(c) = 0$. ✓ Rolles Thm

~~III.~~ $f'(x) > 0$ for $5 < x < 7$.

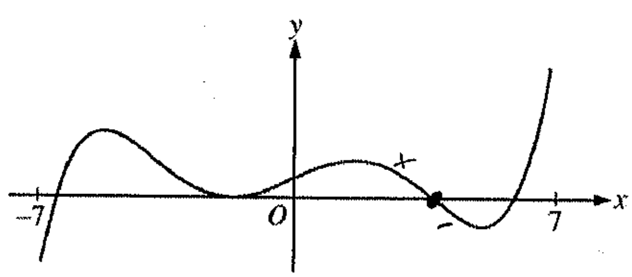
(A) I only

(B) II only

(C) III only

(D) I and III only

(E) I, II, and III



Graph of f'

87. The figure above shows the graph of f', the derivative of the function f, on the open interval $-7 < x < 7$. If f' has four zeros on $-7 < x < 7$, how many relative maxima does f have on $-7 < x < 7$?

(A) One (B) Two (C) Three (D) Four (E) Five

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	–2	–3	4

89. The table above gives values of the differentiable functions f and g and their derivatives at $x = 1$. If $h(x) = (2f(x) + 3)(1 + g(x))$, then $h'(1) =$

(A) –28 (B) –16 (C) 40 $\boxed{\text{(D) } 44}$ (E) 47

90. The functions f and g are differentiable. For all x, $f(g(x)) = x$ and $g(f(x)) = x$. If $f(3) = 8$ and $f'(3) = 9$, what are the values of $g(8)$ and $g'(8)$?

(A) $g(8) = \frac{1}{3}$ and $g'(8) = -\frac{1}{9}$

(B) $g(8) = \frac{1}{3}$ and $g'(8) = \frac{1}{9}$

(C) $g(8) = 3$ and $g'(8) = -9$

(D) $g(8) = 3$ and $g'(8) = -\frac{1}{9}$

$\boxed{\text{(E) } g(8) = 3 \text{ and } g'(8) = \frac{1}{9}}$

$h(x) = (2f(x) + 3)(1 + g(x))$

$h'(x) = (2f(x)+3)(g'(x)) + (1+g(x))(2f'(x))$

$h'(1) = (6+3)(4) + (1-3)(2 \cdot -2)$

$-2 \quad -4$

$h'(1) = 36 + 8$

$f(3) = 8 \quad (3,8) \quad g(8,3) \quad g(8) = 3$

$g'(8) = \frac{1}{f'(3)} = \frac{1}{9}$

(25) $x^2y - 3x = y^3 - 3$

$$x^2(dy/dx) + y(2x) - 3 = 3y^2(dy/dx) \Big|_{(-1,2)}$$

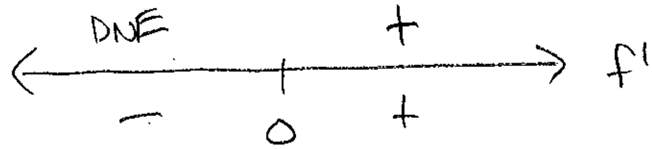
$$1(dy/dx) - 4 - 3 = 12\,dy/dx$$

$$-7 = 11\,dy/dx$$

$$\boxed{\frac{dy}{dx} = \frac{-7}{11}}$$

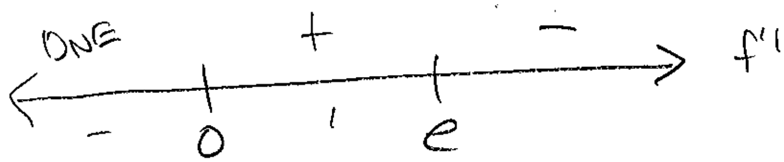
(26) $f'(x) = \dfrac{\ln x}{x}$ $\qquad \ln x \neq 0$

$x = 0$



f' IS INCREASING $(0, \infty)$

$$f''(x) = \frac{1 - \ln x}{x^2}$$



$1 - \ln x = 0 \qquad x^2 = 0$

$e^{\ln x} = e^1 \qquad x = 0$

$x = e$

e POINT OF INFLECTION

$(0, e)$ CONCAVE UP

(e, ∞) CONCAVE DOWN