WARM UP

1.If f(x) is a continuous function for all x, given selected vales of f below, approximate the integral using the trapezoid method.

$$\int_{1}^{11} f(x)dx$$

x	1	2.5	4	6	8	9.5	11
f(x)	3	5	2	7	1	5	8

The FUNdamental Theorem of Calculus Part Two & Mean Value Theorem for Integrals Average Value of a Function

Objective:

- Find the derivative of an integral equation.
- Find the average value of a function on a closed interval.
- Understand and use the MVT for integrals.

Given that $F(x) = \int_1^x \frac{2}{t^2}\,dt$ find $\frac{d}{dx}(F(x))$ **Long Way**

FUNdamental Theorem of Calculus Part 2

$$\frac{d}{dx}\left[\int_{h(x)}^{g(x)} f(t)\,dt\right] = f\big(g(x)\big)\cdot g'(x) - f\big(h(x)\big)\cdot h'(x)$$

Example 1: $\frac{d}{dx}\int_{x^2}^{\frac{1}{x}} t^3 dt$

Example 2: $\frac{d}{dx}\left[\int_5^x \sqrt{p^3}\sin p\, dp\right] =$

Example 3: $\frac{d}{dx}\left[\int_{e^x}^{7}\left(t^2+5t\right)dt\right]$

Example 4:

If $F(x)=\int_{1}^{x} f(t)\,dt$, where $f(t)=\int_{1}^{t^2}\frac{\sqrt{1+u^4}}{u}du$, find $F''(2)$.

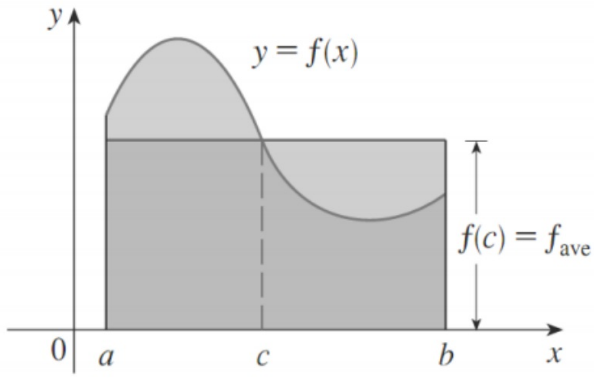
The Mean Value Theorem (for Integrals)

If f is continuous on the closed interval $[a,b]$, then there exists a number $x=c$ in the CLOSED interval $[a,b]$ such that

$$\int_a^b f(x)\,dx = f(c)\cdot(b-a)$$

Where $f(c)$ is called the **average value** of the function f on the interval $[a,b]$. The above equation above can be explicitly solved for $f(c)$.

$$f(c)=\frac{\int_a^b f(x)\,dx}{b-a} \quad \text{or} \quad f(c)=\frac{1}{b-a}\int_a^b f(x)\,dx$$



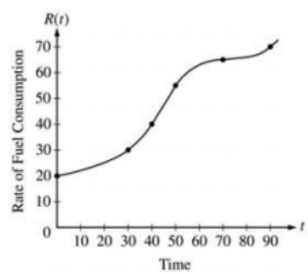
Example 5: Given $f(x) = 1 + x^2$, [-1,2] find:

A. The average value of the function.

B. The value c guaranteed by the MVT for integrals.

Example 6: Find the number(s) b such that the average value of $f(x) = 2 + 6x - 3x^2$ on the interval [0,b] is equal to 3.

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

(a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

(c) Approximate the value of $\int_0^{90} R(t)\,dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)\,dt$? Explain your reasoning.

(d) For $0 < b \le 90$ minutes, explain the meaning of $\int_0^b R(t)\,dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b}\int_0^b R(t)\,dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

(a) $R'(45) \approx \dfrac{R(50) - R(40)}{50 - 40} = \dfrac{55 - 40}{10}$

$= 1.5 \text{ gal/min}^2$

$2: \begin{cases} 1: \text{a difference quotient using numbers from table and interval that contains 45} \\ 1: 1.5 \text{ gal/min}^2 \end{cases}$

(c) $\displaystyle\int_0^{90} R(t)\,dt \approx (30)(20) + (10)(30) + (10)(40) + (20)(55) + (20)(65) = 3700$

Yes, this approximation is less because the graph of R is increasing on the interval.

$2: \begin{cases} 1: \text{value of left Riemann sum} \\ 1: \text{“less” with reason} \end{cases}$

(d) $\displaystyle\int_0^{b} R(t)\,dt$ is the total amount of fuel in gallons consumed for the first b minutes.

$\displaystyle\frac{1}{b}\int_0^{b} R(t)\,dt$ is the average value of the rate of fuel consumption in gallons/min during the first b minutes.

$3: \begin{cases} 2: \text{meanings} \\ \quad 1: \text{meaning of } \int_0^b R(t)\,dt \\ \quad 1: \text{meaning of } \frac{1}{b}\int_0^b R(t)\,dt \\ \quad <-1> \text{ if no reference to time } b \\ 1: \text{units in both answers} \end{cases}$