Advanced Integration Practice Problems

AP Calculus BC

Name: **Answers**

(1) The rate at which a rumor spreads through a high school can be modeled by the differential equation $\frac{dP}{dt} = 0.003P(2000 - P)$, where P is the number of students who have heard the rumor t hours after 9AM.

(a) How many students have heard the rumor when it is spreading the fastest **1000**

(b) If $P(0) = 5$, solve for P as a function of t. $\boldsymbol{P(t) = \frac{2000}{1+399e^{-6t}}}$

(2) $\int \frac{6x^2-x-1}{3x-1}\,dx$ = $\boldsymbol{x^2 + \frac{x}{3} - \frac{2}{9}\ln|3x-1| + C}$

(3) $\int e^{3x} \sin x \, dx$ = $\boldsymbol{\frac{1}{10}(3e^{3x}\sin x - e^{3x}\cos x) + C}$

(4) $\lim_{x\to\infty}\left(1+\frac{4}{x}\right)^x$ = $\boldsymbol{e^4}$

(5) $\lim_{x\to\frac{\pi}{2}}(\sec x - \tan x)$ = **0**

(6) $\int (x^2 - 5x)e^x dx$ = $\boldsymbol{e^x(x^2 - 7x + 7) + C}$

(7) $\int_0^{\pi/2} \tan\theta \, d\theta$ **Diverges**

(8) The populator of $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt} = P\left(2 - \frac{P}{5000}\right)$, where the initial population is $P(0) = 3000$ and t is the time in years. What is $\lim_{t\to\infty} P(t)$?

(A) 2500 (B) 3000 (C) 4200 (D) 5000 **(E) 10,000**

(9) Which of the following startements about the integral $\int_0^{\pi} \sec^2 x dx$ is true?

(A) The integral is equal to 0

(B) The integral is equal to $\frac{2}{3}$

(C) The integral diverges because $\lim_{x\to\frac{\pi}{2}^-} \sec^2 x$ does not exist.

(D) The integral diverges because $\lim_{x\to\frac{\pi}{2}^-} \tan x$ does not exist

(10) $\int_{-1}^{1} \frac{e^x}{e^x-1}\,dx$ **Diverges**

(11) $\int_{e}^{\infty} \frac{1}{x(\ln x)^2}\,dx = \mathbf{1}$

(12) If $\int f(x)\sin x\,dx = -f(x)\cos x + \int 3x^2\cos x\,dx$, then $f(x)$ could be

(A) $3x^2$ **(B) x^3** (C) $-x^3$ (D) $\sin x$ (E) $\cos x$

(13) The function N satisfies the logistic differential equation $\frac{dN}{dt} = \frac{N}{10}\left(1 - \frac{N}{850}\right)$, where $N(0)$=105. Which of the following statements is false?

(A) $\lim_{t\to\infty} N(t) = 850$

(B) $\frac{dN}{dt}$ has a maximum value when $N = 105$

(C) $\frac{d^2N}{dt^2} = 0$ when $N = 425$.

(D) When $N > 425, \frac{dN}{dt} > 0$ and $\frac{d^2N}{dt^2} < 0$.

(14) Find the value of the area under the curve of $y = \frac{1}{x^2+1}$ in the first quadrant? $\boldsymbol{A = \frac{\pi}{2}}$

(15) $\int_{-1}^{\infty} \frac{1}{x^2+5x+6}\,dx = \mathbf{\ln 2}$

(16) $\int x^2 \ln x\,dx = \boldsymbol{\frac{x^3 \ln x}{3} - \frac{1}{9}x^3 + C}$

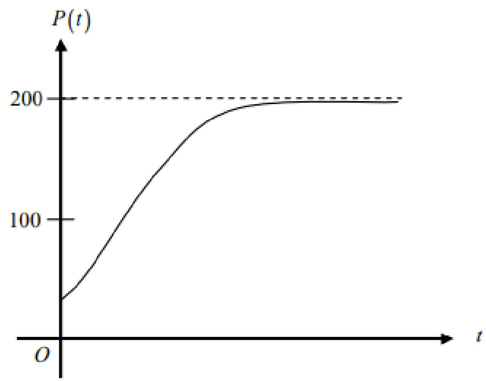
(17) Let R be the region between the graph of $y = e^{-2x}$ and the x-axis for $x \geq 3$. The area of R is

(A) $\boldsymbol{\frac{1}{2e^6}}$ (B) $\frac{1}{e^6}$ (C) $\frac{2}{e^6}$ (D) $\frac{\pi}{2e^6}$ (E) infinite

(18) $\int_{0}^{1} \frac{5x+8}{x^2+3x+2}\,dx$ is

(A) $\ln(8)$ (B) $\ln(\frac{27}{2})$ **(C) $\ln(18)$** (D) $\ln(288)$ (E) divergent

(19) $\lim_{x\to 0^+} (\sin x)^x = \mathbf{1}$



(20) Which of the following differential equations for a population P could model the logistic growth shown in the figure above?

(A) $\boldsymbol{\frac{dP}{dt} = 0.2P - 0.001P^2}$ (B) $\frac{dP}{dt} = 0.1P - 0.001P^2$ (C) $\frac{dP}{dt} = 0.2P^2 - 0.001P$

(D) $\frac{dP}{dt} = 0.1P^2 - 0.001P$ (E) $\frac{dP}{dt} = 0.1P^2 + 0.001P$