Polar Derivatives

Objective: Find the derivative of polar equations.

Given the derivative formula for a polar equation, determine an equivalent form for the polar derivatives.

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

Derivative of a Polar Equation

To find the slope of a tangent line to a polar graph $r = f(\theta)$, we can use $x = r\cos\theta$ and $y = r\sin\theta$ and the product rule:

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{r\cos\theta + r'\sin\theta}{-r\sin\theta + r'\cos\theta}, \text{ provided that } \frac{dx}{d\theta} \neq 0$$

Or .... create $y(\theta)$ and $x(\theta)$ first, then find $y'(\theta)$ and $x'(\theta)$.

Find the equation of line tangent to the polar curve $r = \sin 2\theta$ when $\theta = 3\pi/4$. (No Calculator)

Consider the cardiod $r = 1 + \sin\theta$. For $0 \leq \theta \leq 2\pi$, determine the values where the curve has a horizontal and vertical tangents. Be careful of 0/0. (Calculator Active)

A curve is described by the equation in polar coordinates $r = \theta^2 - 2\cos(2\theta)$ for $0 \leq \theta \leq \pi$, where r is measured in feet and θ is measured in radians. (Calculator Active)

(a) Determine x-value at which the curve is furthest to the right.

(b) Find the area bounded above the x-axis.

(c) Describe the motion of the particle at $\theta = \pi/6$ in relation to the origin. At $\theta = 2\pi/3$?