WARM UP

1. The line y = -7 is a horizontal asymptote to the graph of which of the following functions?

(A) $y = -\frac{\sin(7x)}{x}$ (B) $y = \frac{-7x^2 + 2x - 1}{\sqrt{x^2 + 50}}$ (C) $y = \frac{1}{x+7}$

(D) $y = \frac{21x^3 - 2x^2 - 7}{7 + 9x - 3x^3}$ (E) $y = \frac{-7x}{1-x}$

2. If $f(x) = \cot x$ and $\frac{3}{y} = f(x)$, find $\left.\frac{dy}{dx}\right|_{x=\frac{11\pi}{6}}$

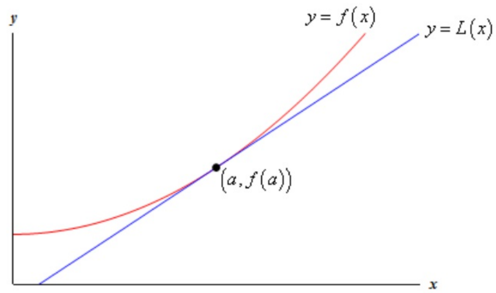
(A) 0 (B) 4 (C) 12 (D) $3\sqrt{3}$ (E) $-\sqrt{3}$

LINEARIZATION AND TANGENT-LINE APPROXIMATIONS

Objective:

- Use the tangent line to make approximations for a function.
- Use concavity to determine if it is an over-approximation or an under-approximation.

If f(x) is differentiable at x=c, then we say it is locally linear at x=c. This means that as we zoom in closer and closer around x=c, the graph of f(x) will begin to look more and more like the tangent line at x=c.



$$y - f(a) = f\,'(x)(x - a)$$

$$L(x) = f(a) + f\,'(x)(x - a)$$

Steps for linear approximation:

1. Find the equation of the tangent line at the given point.

2. Use the tangent line to make your approximation. Be sure to use approximation symbol: $\approx$

3. If asked, use concavity to determine under/over approximation:
 - If f "(c) < 0, f(x) is concave down and its over
 - If f "(c) > 0, f(x) is concave up and its under

Example 1: Find the local linear approximation of $f(x) = x^3 - 2x + 3$ at the point where $x = 2$.

Use your approximation to estimate

A. $f(2.1)$

B. $f(1.9)$

Example 2: Use the line tangent to $f(x) = \frac{3}{(1-x)^2}$ at $x=0$ to approximate the value of $x=0.05$. Is it an over/under approximation?

Example 3: Let f be a differentiable function such that $f(3) = 2$ and $f'(3) = 5$. If the tangent line to the graph of f at $x=3$ is used to find an approximation to a zero of f, that approximation is:

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Example 4: If $f(1) = 2$ and $dy/dx = xy^3$ and $d^2y/dx^2 = y^3(1+3x^2y^2)$

A. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.

B. Use the tangent line equation from part (A) to approximate $f(1.1)$. Given that $f(x) > 0$ for $1 < x < 1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain

Example 5:

. The function f is continuous on the closed interval [2, 4] and twice differentiable on the open interval (2, 4). If $f'(3) = 2$ and $f''(x) < 0$ on the open interval (2, 4), which of the following could be a table of values for f?

(A)

x	$f(x)$
2	2.5
3	5
4	6.5

(B)

x	$f(x)$
2	2.5
3	5
4	7

(C)

x	$f(x)$
2	3
3	5
4	6.5

(D)

x	$f(x)$
2	3
3	5
4	7

(E)

x	$f(x)$
2	3.5
3	5
4	7.5