WARM UP

1. $\int_{1}^{\infty} \frac{2+x}{x^2}\,dx$

2. Let *f* be a differentiable function such that *f(3)* = 2 and *f ' (3)* = 5. If the tangent line to the graph of f at x = 3 is used to find an approximation to a zero of *f*, that approximation is

(a) 0.4 (b) 0.5 (c) 2.6 (d) 3.4 (e) 5.5

Improper Integrals
(Day 2)

Objective:

- Evaluate improper integrals that have an infinite discontinuity on the interval of integration or show that the integral diverges.

Evaluating with one sided limits

Definition of Improper Integrals with Infinite Discontinuities

1. If f is continuous on the interval $[a, b)$ and has an infinite discontinuity at b, then

$$\int_a^b f(x)\,dx = \lim_{c \to b^-} \int_a^c f(x)\,dx.$$

2. If f is continuous on the interval $(a, b]$ and has an infinite discontinuity at a, then

$$\int_a^b f(x)\,dx = \lim_{c \to a^+} \int_c^b f(x)\,dx.$$

3. If f is continuous on the interval $[a, b]$, except for some c in (a, b) at which f has an infinite discontinuity, then

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx.$$

In the first two cases, the improper integral **converges** when the limit exists—otherwise, the improper integral **diverges**. In the third case, the improper integral on the left diverges when either of the improper integrals on the right diverges.

Example 1: $\int_0^{27} \frac{dx}{\sqrt[3]{27 - x}}$

Example 2: $\int_0^3 \frac{dx}{(x-1)^{\frac{2}{3}}}$

Example 3: $\int_0^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$