WARM UP

1. Let f be a function such that $\lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = 5$. Which of the following must be true?

I. f is continuous at $x = 2$.

II. f is differentiable at $x = 2$.

III. The derivative of f is continuous at $x = 2$.

(A) I only (B) II only (C) I and II only (D) I and III only (E) II and III only

2.

Let f be the function given by $f(x) = 2e^{4x^2}$. For what value of x is the slope of the line tangent to the graph of f at $(x, f(x))$ equal to 3?

(A) 0.168 (B) 0.276 (C) 0.318 (D) 0.342 (E) 0.551

Ratio and Root Test

Objective:

- Use the rato and root test to determine if a series converges.

THEOREM 9.17 Ratio Test

Let Σa_n be a series with nonzero terms.

1. The series Σa_n converges absolutely when $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$.
2. The series Σa_n diverges when $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| > 1$ or $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \infty$.
3. The Ratio Test is inconclusive when $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$.

This test works well for factorials and/or exponentials.

(a) $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

(b) $\sum_{n=1}^{\infty} \frac{n^2 3^{n+1}}{2^n}$

(c) $\sum_{n=1}^{\infty} \frac{(n+1)!}{3^n}$

THEOREM 9.18 Root Test

1. The series Σa_n converges absolutely when $\lim_{n\to\infty} \sqrt[n]{|a_n|} < 1$.

2. The series Σa_n diverges when $\lim_{n\to\infty} \sqrt[n]{|a_n|} > 1$ or $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \infty$.

3. The Root Test is inconclusive when $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$.

Note: This is usually the best test when the whole series can be written to a power of n.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$

(b) $\sum_{n=1}^{\infty}\left(\frac{3n+4}{2n}\right)^n$

GUIDELINES FOR TESTING A SERIES FOR CONVERGENCE OR DIVERGENCE

1. Does the nth term approach 0? If not, the series diverges.
2. Is the series one of the special types—geometric, p-series, telescoping, or alternating?
3. Can the Integral Test, the Root Test, or the Ratio Test be applied?
4. Can the series be compared favorably to one of the special types?

Determine the convergence or divergence of each series.

a. $\sum_{n=1}^{\infty} \frac{n+1}{3n+1}$

b. $\sum_{n=1}^{\infty} \left(\frac{\pi}{6}\right)^n$

c. $\sum_{n=1}^{\infty} ne^{-n^2}$

d. $\sum_{n=1}^{\infty} \frac{1}{3n+1}$

e. $\sum_{n=1}^{\infty} (-1)^n \frac{3}{4n+1}$

f. $\sum_{n=1}^{\infty} \frac{n!}{10^n}$

g. $\sum_{n=1}^{\infty} \left(\frac{n+1}{2n+1}\right)^n$

(a) $\sum_{n=1}^{\infty} \frac{1+3n^2+n^3}{4n^3-5n+2}$

(b) $\sum_{n=0}^{\infty} \left(\frac{2}{7}\right)^n$

(c) $\sum_{n=1}^{\infty} \frac{4}{n^3}$

(d) $\sum_{n=1}^{\infty} \frac{n^2}{5^n}$

(e) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^5+5}}$

(f) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^4}$

(g) $\sum_{n=1}^{\infty} \frac{5n^2-6n+3}{n^3-7n+8}$

(h) $\sum_{n=1}^{\infty} \frac{\cos n\pi}{\sqrt{n}}$

(i) $\sum_{n=1}^{\infty} \frac{3^n+4}{2^n}$

(j) $\sum_{n=1}^{\infty} \frac{8n^3-6n^5}{12n^4-9n^5}$

(k) $\sum_{n=1}^{\infty} \sqrt{\frac{3n+1}{n^5+2}}$

(l) $\sum_{n=1}^{\infty} \frac{3^{n-1}}{n2^n}$

(m) $\sum_{n=1}^{\infty} \left(\frac{2n}{5n-1}\right)^n$

(n) $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^n}$

(o) $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^n$

(p) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln n}$