WARM UP

The function f is defined by the power series

$$f(x) = -\frac{x}{2} + \frac{2x^2}{3} - \frac{3x^3}{4} + \cdots + \frac{(-1)^n nx^n}{n+1} + \cdots$$

for all real numbers x for which the series converges. The function g is defined by the power series

$$g(x) = 1 - \frac{x}{2!} + \frac{x^2}{4!} - \frac{x^3}{6!} + \cdots + \frac{(-1)^n x^n}{(2n)!} + \cdots$$

for all real numbers x for which the series converges.

(a) Find the interval of convergence of the power series for f. Justify your answer.

(b) The graph of $y = f(x) - g(x)$ passes through the point $(0, -1)$. Find $y'(0)$ and $y''(0)$. Determine whether y has a relative minimum, a relative maximum, or neither at $x = 0$. Give a reason for your answer.

Power Series - Part II More fun with Power Series

Objective:

- Represent power series using geometric series not centered at zero.
- Use power series to create new power series.
- Find an infinite sum by recognizing a convering Taylor's Series.

Ex.1: Find a power series centered at x = 1.
Find first four nonzero terms and general term.

$$f(x) = \frac{1}{x}$$

Ex.2: Find a power series centered at x = 1.
Find first four nonzero terms and general term.
(use geometric series)

$$f(x) = \frac{1}{x}$$

Ex.3: How could we use 1/x to find power series for lnx centered at x = 1?

Ex.4: Find a power series that represents

$$\frac{1}{(1-x)^2}$$

Hint: $\frac{d}{dx}\left[\frac{1}{1-x}\right]$

You can use converging Taylor series to find infinite sum evaluated at a particular value of x.

$$1+\frac{2}{1!}+\frac{4}{2!}+\frac{8}{3!}+\cdots+\frac{2^n}{n!}+\cdots$$

$$1-\frac{1}{3!}+\frac{1}{5!}-\frac{1}{7!}+\cdots+\frac{(-1)^n}{(2n+1)!}+\cdots$$

Let f be the function given by $f(x)=e^{-x^2}$.

(a) Write the first four nonzero terms and the general term of the Taylor series for f about $x=0$.

(b) Use your answer to part (a) to find $\lim_{x\to 0}\frac{1-x^2-f(x)}{x^4}$.

(c) Write the first four nonzero terms of the Taylor series for $\int_0^x e^{-t^2}\,dt$ about $x=0$. Use the first two terms of your answer to estimate $\int_0^{1/2} e^{-t^2}\,dt$.

(d) Explain why the estimate found in part (c) differs from the actual value of $\int_0^{1/2} e^{-t^2}\,dt$ by less than $\frac{1}{200}$.