WARM UP - Calculator Inactive

1. A population of animals is modeled by a function P that satisfies the logistic differential equation $dP/dt = 1P - .01P^2$, where t is measured in years.

A. If $P(0) = 20$, solve for P as a function of t.

B. $\lim_{t \to \infty} P(t) =$

Improper Integrals

(Day 1)

Objective:

- Evaluate an improper integral or show that an improper integral diverges.

Improper integrals are improper because:

A. They have an infinite interval of integration
B. They have a discontinuity on the interval of integration
or both A & B.

They are evaluated by rewriting the integral as a proper integral and then using limits. The limit notatin is very important!

If the limit equals a finite value then the integral converges to that value. If the limit does not equal a finite number the integral diverges.

Definition of Improper Integrals with Infinite Integration Limits

1. If f is continuous on the interval $[a, \infty)$, then

$$\int_a^{\infty} f(x)\,dx = \lim_{b \to \infty} \int_a^b f(x)\,dx.$$

2. If f is continuous on the interval $(-\infty, b]$, then

$$\int_{-\infty}^{b} f(x)\,dx = \lim_{a \to -\infty} \int_a^b f(x)\,dx.$$

3. If f is continuous on the interval $(-\infty, \infty)$, then

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{c} f(x)\,dx + \int_c^{\infty} f(x)\,dx$$

where c is any real number (see Exercise 111).

In the first two cases, the improper integral **converges** when the limit exists—otherwise, the improper integral **diverges**. In the third case, the improper integral on the left diverges when either of the improper integrals on the right diverges.

Ex.1: $\int_{1}^{\infty} e^{-x}\,dx$

Ex. 2: $\int_{1}^{\infty} \frac{1}{x^{2/3}}\,dx$

Ex. 3: $\int_0^{\infty} xe^{-x}dx$

Ex. 4: $\int_{-1}^{\infty} \frac{dx}{x^2 + 5x + 6}$

Ex. 5: $\int_{-\infty}^{\infty} xe^{-x^2}\,dx$