Warm-up

1. If $f(x) = x - \frac{1}{x}$ find f ''(x)

(2) For the equation $f\left(x\right) = \frac{x}{x+1}$ find:

a. f '(1)

b. Equation of the normal line at x = 1.

Velocity and Other Rates of Change

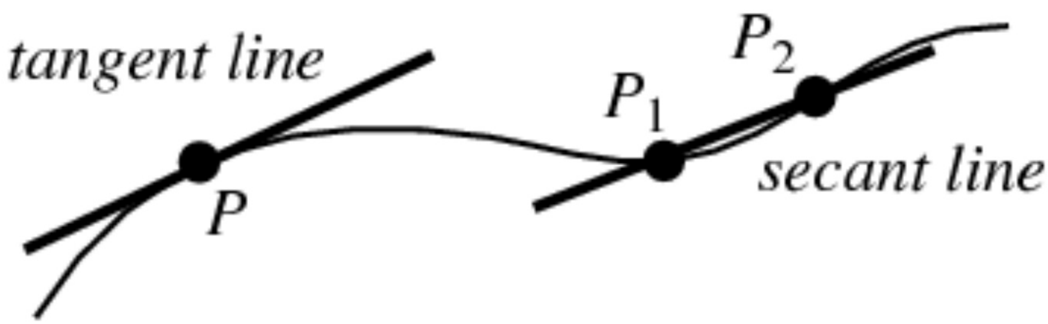
Objective:

- Find and evaluate rates of change
- Analyze particle motion

Average vs Instanteous Rate of Change

Average rate of change is the change over a given time interval (time). *Algebra Slope*

Instaneous rate of change is how fast an particle is change a specific time. *Calculus Slope*



Important - Big Idea!

The derivative of a function can be interpreted as the instantaneous rate of change with respect to its independent variable.

The function can represent any quantity, such as velocity, temperature of an oven (or potato), area or volume, etc...

At time $t = 0$, a boiled potato is taken from a pot on a stove and left to cool in a kitchen. The internal temperature of the potato is 91 degrees Celsius (°C) at time $t = 0$, and the internal temperature of the potato is greater than 27°C for all times $t > 0$. The internal temperature of the potato at time t minutes can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = -\frac{1}{4}(H - 27)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 91$.

Ex.1: The temperature T, in degrees Fahrenheit, of a potato placed in a hot oven is given by $T = f(t)$, where t is the time in minutes since the potato was put in the oven.

(a) Describe the meaning of $f(20) = 255$.

(b) What is the sign of $f'(t)$? Why?

(c) What are the units of $f'(20)$?

(d) Explain the meaning of $f'(20) = 2$.

Position - Original Function (units)

f(x)   s

Velocity - First Derivative (units/time)

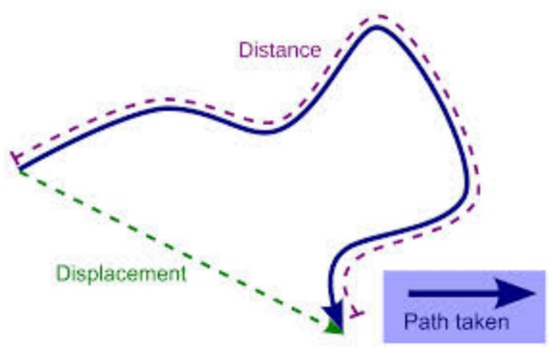
f'(x) $$\frac{ds}{dt} = v$$

Acceleration - Second Derivative (units/time^2)

f''(x) $$\frac{d^2s}{dt^2} = \frac{dv}{dt} = a$$

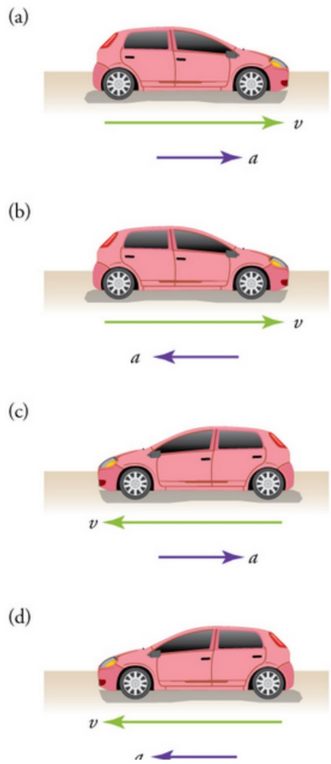
Other Volcabulary

Displacement: Distance from point of origin to final position.

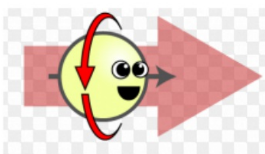


Speed: Absolute value of velocity. *ALWAYS POSITIVE*

Speeding Up or Slowing Down?



Ex.2: Particle motion



A particle moves along a line so that its position at any given time $t \geq 0$ is given by the function $s(t) = t^2 - 4t + 3$, where s is measured in meters and t is measured in seconds.

a) Find the displacement of the particle during the first 2 seconds.

b) Find the average velocity of the particle during the first 4 seconds.

$s(t) = t^2 - 4t + 3$

c) Find the instantaneous velocity of the particle when $t = 4$.

d) Find the acceleration of the particle at $t = 4$.

e) Describe the motion of the particle. At what value of t does the particle change direction?

Ex.3: A particle moves along the x-axis in such a way that its position at time t for $t>0$ is given by:

$$x = \frac{1}{3}t^3 - 3t^2 + 8t$$

A. Show at $t=0$, the particle is moving to the right.

B. Find all the values of t for which the particle is moving to the left.

$$x = \frac{1}{3}t^3 - 3t^2 + 8t$$

C. Determine if the particle is speeding up or slowing down at the given times.

(i) $t = 1$ (ii) $t = 5$