WARM UP

1. $\lim_{x \to 3} \dfrac{\sqrt{x+1} - 2}{x - 3}$

2. Is the function continuous at x=2? $f(x) = \begin{cases} x+7, x < 2 \\ 9, x = 2 \\ 3x+3, x > 2 \end{cases}$

3. Use the IVT to prove that the function $f(x) = x^2 - 2 - \cos x$ has a zero on the interval $[0, \pi]$.

THEOREM 1.2 Properties of Limits

Let b and c be real numbers, let n be a positive integer, and let f and g be functions with the limits

$$\lim_{x \to c} f(x) = L \quad \text{and} \quad \lim_{x \to c} g(x) = K.$$

1. Scalar multiple: $\lim_{x \to c} [b f(x)] = bL$
2. Sum or difference: $\lim_{x \to c} [f(x) \pm g(x)] = L \pm K$
3. Product: $\lim_{x \to c} [f(x)g(x)] = LK$
4. Quotient: $\lim_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{L}{K}, \quad K \neq 0$
5. Power: $\lim_{x \to c} [f(x)]^n = L^n$

THEOREM 1.5 The Limit of a Composite Function

If f and g are functions such that $\lim_{x \to c} g(x) = L$ and $\lim_{x \to L} f(x) = f(L)$, then

$$\lim_{x \to c} f(g(x)) = f\left(\lim_{x \to c} g(x)\right) = f(L).$$

Example 1:

Given $\lim_{x\to3} f(x)=8$, $\lim_{x\to3} g(x)=-2$, and $\lim_{x\to3} h(x)=0$, find the limits that exist.

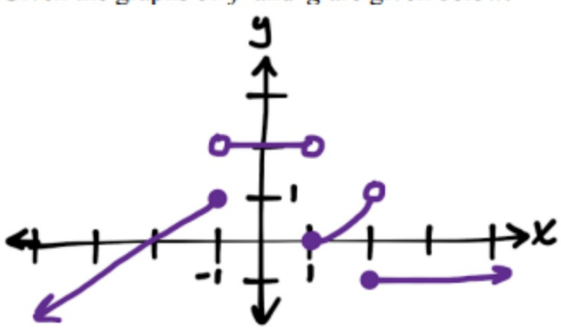
(a) $\lim_{x\to3}\left[2f(x)-4g(x)\right]=$

(b) $\lim_{x\to3}\left[2g(x)\right]^2=$

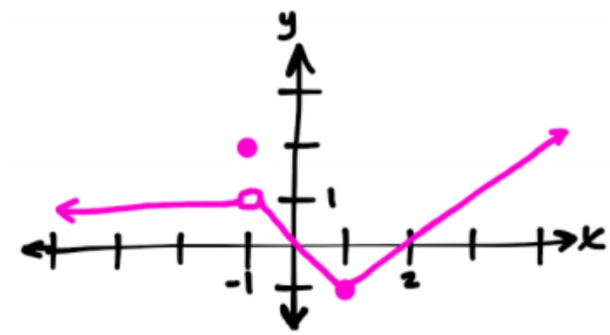
(c) $\lim_{x\to3}\left(\frac{\sqrt[3]{f(x)}}{g(x)}+\frac{4h(x)}{x+7}\right)=$

Example 2:

Given the graphs of f and g are given below.



graph of f



graph of g

Determine whether the following limits exist. If they do, then find the limit.

(a) $\lim_{x\to0}\left[2[f(x)]^2+3g(x)=\right]$

(b) $\lim_{x\to-3}\frac{x^2g(x)}{f(x)}=$

(c) $\lim_{x\to-3} g(f(x))=$

(d) $\lim_{x\to-1} g\left(x^2\right)=$

(e) $\lim_{x\to1^-}\frac{2f(x)}{[g(x)]^2}=$

(f) $\lim_{x\to-1}\left(5-\sqrt{x^2\left(3+f(x)\right)}\right)=$