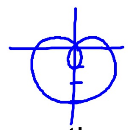
WARM UP - Calculator Inactive

$u = \tan x$

$du = \sec^2 x\,dx$

1. $\int 5\sec^2 x \cdot e^{\tan x} dx = 5\int e^u du$

$= 5e^{\tan x} + C$



2. Find the area of the region between the inner and outer loop of r = 1 - 2sinθ (set-up only)

$0 = 1 - 2\sin\theta$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\frac{1}{2}\int_{\frac{5\pi}{6}}^{\frac{13\pi}{6}} r^2 d\theta - \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} r^2 d\theta$

Go over your homework with a partner.

Alternating Series Test

Objective:

- Determine if an Alternating Series converges.
- Approximate the sum of the series using the *nth* partial sum and determine the error.
- Determine if a series converges conditionally or converges absolutely.

Alternating Series - a series whose terms are alternately positive and negative on consecutive terms.

For example: $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$

Alternating Series Test (AST)

If $a_n > 0$, then the alternating series $\sum_{n=1}^{\infty} (-1)^n a_n$ or $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ converges if both of the following conditions are satisfies:

1) $\lim_{n\to\infty} a_n = 0$

2) $\{a_n\}$ is a decreasing (or Non-increasing) sequence; that is, $a_{n+1} \le a_n$ for all $n > k$, for some $k \in \mathbb{Z}$

Note: This test only proves convergences!

(a) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{2n-1}$

$\lim_{n\to\infty} \frac{n}{2n-1} = \frac{1}{2} \neq 0$ (1) not satisfied

diverges by nth term test

$\lim_{n\to\infty} \frac{(-1)^{n+1} n}{2n-1}$ diverges by oscillation.

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{\ln(2n)}$

(1) $\lim_{n\to\infty} \frac{n}{\ln(2n)} =$

$= \lim_{n\to\infty} \frac{1}{\frac{1}{\cancel{2}n}\cdot\cancel{2}} = n = \infty$

= diverges by nth term test.

(c) $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

(1) $\lim_{n\to\infty} \frac{1}{n} = 0$ ✓

(2) $\frac{1}{n+1} < \frac{1}{n}$ ✓ decreases.

→ converge by Alt Series Test

→ (conditionally converges

you'll learn this later.

$$\sum_{n=1}^{\infty} \frac{n}{(-2)^{n-1}} = \sum_{n=1}^{\infty} \left(-\frac{1}{2}\right)^{n-1} \cdot n$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n-1} n}{2^{n-1}} = \frac{(-1)^{n-1} 2n}{2^n}$$

(1) $\lim_{n\to\infty} \frac{2n}{2^n} = 0$ and (2) decreases

∴ Series converges by AST.

Absolute vs Conditional Convergence

THEOREM 9.16 Absolute Convergence

If the series $\Sigma |a_n|$ converges, then the series Σa_n also converges.

Definitions of Absolute and Conditional Convergence

1. The series Σa_n is **absolutely convergent** when $\Sigma |a_n|$ converges.
2. The series Σa_n is **conditionally convergent** when Σa_n converges but $\Sigma |a_n|$ diverges.

Determine whether the given series converges or diverges. If it converges, determine whether it is absolutely convergent or conditionally convergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n}$$

(1) $\lim_{n\to\infty} \frac{1}{2^n} = 0$ ✓

(2) $\frac{1}{2^n} > \frac{1}{2^{n+1}}$ ✓

converges by AST

$\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$

converges by GST because $|r| < 1$

$\therefore$ absolutely converges (with or without the alternator)

$$\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n+1}$$

1

(1) $\lim_{n\to\infty} \frac{\sqrt{n}}{n+1} = 0$ ✓

(2) $\frac{\sqrt{n}}{n+1} < \frac{\sqrt{n+1}}{n+2}$ decreasing

converges by AST

2) compare to $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ $p < 1$ diverges by P-series

3) $\lim_{n\to\infty} \frac{\sqrt{n}}{n+1} \cdot \frac{\sqrt{n}}{1} = 1$

diverges by LCT

$\therefore \sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n+1}$ conditionally converges (alternator causes convergence)

Alternate Series Remainder

Suppose an alternating series satisfies the conditions of the AST, namely that $\lim_{n\to\infty} a_n = 0$ and $\{a_n\}$ is not increasing. If the series has a sum S, then $|R_n| = |S - S_n| \le a_{n+1}$, where S_n is the nth partial sum of the series.

In other words, if an alternating series satisfies the conditions of the AST, you can approximate the sum of the series by using the nth partial sum, S_n, and your error will have an absolute value no greater than the first term left off, a_{n+1}. This means $S \in [S_n - R_n, S_n + R_n]$

$|R_n| = a_{n+1}$ → next term

$\underbrace{S_n - R_n}_{\text{lower}} \le S \le \underbrace{S_n + R_n}_{\text{upper}}$

the actual sum

Approximate the sum by using the first six terms, and find the error and the interval in which S must lie.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n!}$$

$S_6 = 91/144 = .631944$

$R_6 = \frac{(-1)^{7-1}}{7!} = \frac{1}{5040} = .0000198$

$.6317 \le S \le .6324$

Approximate the sum with an error less than 0.001.

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^4} = \left[1 - \frac{1}{2^4} + \frac{1}{3^4} - \frac{1}{4^4} + \frac{1}{5^4}\right] - \frac{1}{6^4}$$

.012 .003 .0016 .000007

$S_5 = .9475$

TICKET OUT

1. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}5}{n}$

2. $\displaystyle\sum_{n=1}^{\infty}\frac{2^n}{4n^2-1}$

3. $\displaystyle\sum_{n=1}^{\infty}\frac{2n^2-1}{3n^5+2n+1}$

4. $\displaystyle\sum_{n=2}^{\infty}\frac{\ln n}{n}$