WARM UP - Calculator Inactive

1. $\int 5\sec^2 x \cdot e^{\tan x} dx =$

2. Find the area of the region between the inner and outer loop of r = 1 - 2sinθ (set-up only)

Alternating Series Test

Objective:

- Determine if an Alternating Series converges.
- Approximate the sum of the series using the *nth* partial sum and determine the error.
- Determine if a series converges conditionally or converges absolutely.

Alternating Series - a series whose terms are alternately positive and negative on consecutive terms.

For example: $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$

Alternating Series Test (AST)

If $a_n > 0$, then the alternating series $\sum_{n=1}^{\infty}(-1)^n a_n$ or $\sum_{n=1}^{\infty}(-1)^{n+1} a_n$ converges if both of the following conditions are satisfies:

1) $\lim_{n\to\infty} a_n = 0$

2) $\{a_n\}$ is a decreasing (or Non-increasing) sequence; that is, $a_{n+1} \le a_n$ for all $n > k$, for some $k \in \mathbb{Z}$

Note: This test only proves convergences!

(a) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1} n}{2n-1}$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{\ln(2n)}$

(c) $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$

$$\sum_{n=1}^{\infty} \frac{n}{(-2)^{n-1}}$$

Absolute vs Conditional Convergence

THEOREM 9.16 Absolute Convergence

If the series $\Sigma\,|a_n|$ converges, then the series $\Sigma\, a_n$ also converges.

Definitions of Absolute and Conditional Convergence

1. The series $\Sigma\, a_n$ is **absolutely convergent** when $\Sigma\,|a_n|$ converges.
2. The series $\Sigma\, a_n$ is **conditionally convergent** when $\Sigma\, a_n$ converges but $\Sigma\,|a_n|$ diverges.

Determine whether the given series converges or diverges. If it converges, determine whether it is absolutely convergent or conditionally convergent.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

$$\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n+1}$$

Alternate Series Remainder

Suppose an alternating series satisfies the conditions of the AST, namely that $\lim_{n\to\infty} a_n = 0$ and $\{a_n\}$ is not increasing. If the series has a sum S, then $|R_n| = |S - S_n| \le a_{n+1}$, where S_n is the nth partial sum of the series.

In other words, if an alternating series satisfies the conditions of the AST, you can approximate the sum of the series by using the nth partial sum, S_n, and your error will have an absolute value no greater than the first term left off, a_{n+1}. This means $S \in [S_n - R_n, S_n + R_n]$

Approximate the sum by using the first six terms, and find the error and the interval in which S must lie.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n!}$$

Approximate the sum with an error less than 0.001.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4}$$