WARM UP

1. Let f be the function defined by $f(x) = xe^{1-x}$ for all real numbers x. Find each interval on which f is increasing.

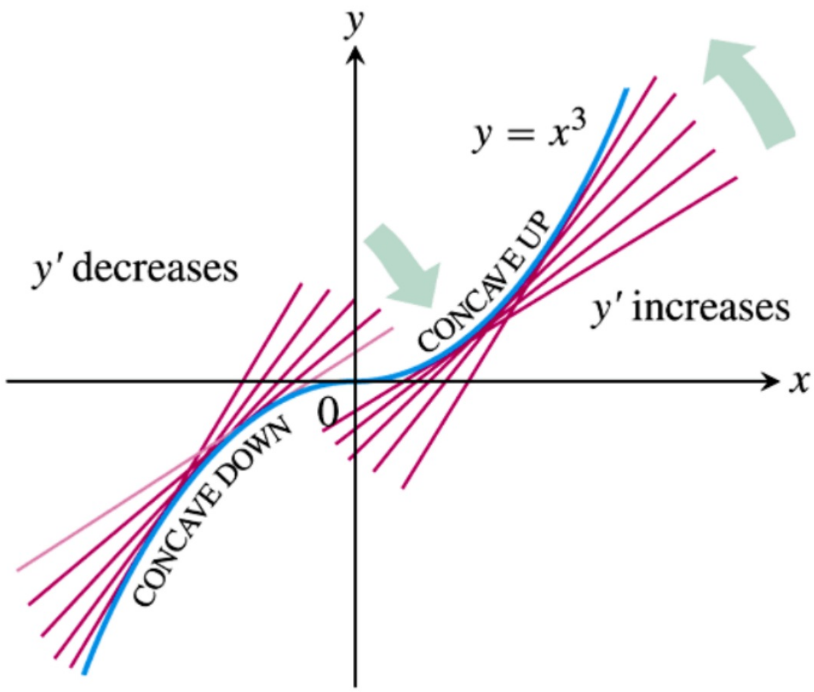
2. Consider the curve defined by the equation $y + \cos y = x + 1$ for $0 \leq y \leq 2\pi$. Write an equation for each vertical tangent to the curve.

Second Derivative Test and Concavity

Objective:

Determine intevals of concavity using the second second derivative test.

Concavity



Def. of Concavity

The graph of a differentiable function $y = f(x)$ is

(a) concave up on an open interval I if y' is increasing on I.

(b) concave down on an open interval I if y' is decreasing on I.

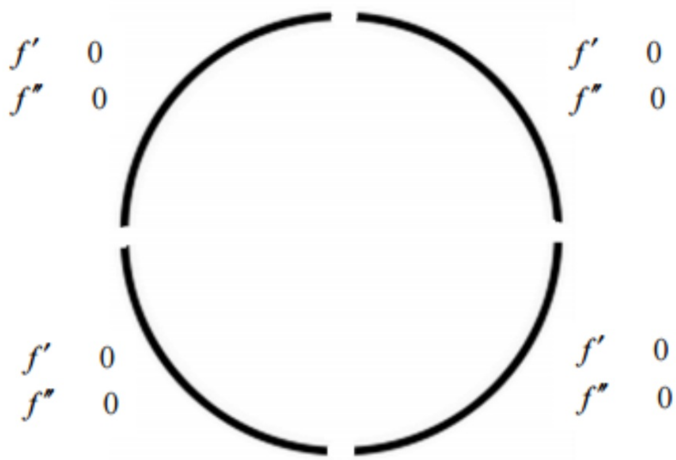
Concavity Test

The graph of a twice-differentiable function $y = f(x)$ is

(a) concave up on an open interval where $y'' > 0$.

(b) concave down on an open interval where $y'' < 0$.

Determine the signs of f' and f'' for each of the curved segments below. Fill in the inequality.



Example 1: Determine the intervals where the graph is concave up/down. Justify your answer.

$$f(x) = x^4 - 4x^3$$

Point of Inflection

An x-value, $x = p$, in the domain of a function $f(x)$, is said to be an **inflection value** of $f(x)$ if the graph of $f(x)$ changes from either concave up to concave down at $x = p$ or from concave down to concave up at $x = p$.

That is, either f'' changes from positive to negative at $x = p$ or f'' changes from negative to positive at $x = p$.

The point $(p, f(p))$ is called the **inflection point**.

Possible inflection values is where either f "(x) = 0 or f "(x) = DNE, the critical values of f '.

Point of Inflection

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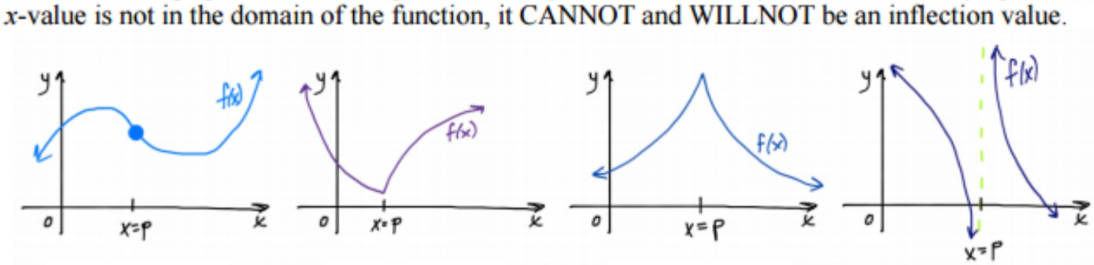
Important note: A graph can also change its concavity at a **discontinuity**, like a vertical asymptote, but if this x-value is not in the domain of the function, it CANNOT and WILLNOT be an inflection value.



Inflection value at $x = p$ | Inflection value at $x = p$ | NO inflection value at $x = p$ | NO inflection value at $x = p$

Example 2: Find all points of inflection. Justify

$$y = e^{-x^2}$$

Example 3: Find all points of inflection. Justify.

$$g(x) = \frac{x^2 + 1}{x^2 - 4}$$

Second Derivative Test for Local Extrema

1. If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at $x = c$.
2. If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at $x = c$.

Example 4: Use the second derivative test to find the x-values for the local extreme values of $f(x) = x^3 - 6x + 5$.

Example 5: Selected values for a twice-differentiable function f(x), continuos on, $-3 \leq x \leq 5$ is given with selected values for f '(x) and f "(x)

x	$f(x)$	$f'(x)$	$f''(x)$
-3	0	5	1
2	2	0	4
5	7	-1	0

A. Does f(x) have a local min or max at x = 2? Justify

B. Explain why there must be a number z, such that f(z) = 6.

Classwork

(1) Determine the open intervals on which the graph is concave upward or concave downward. $f(x) = \dfrac{2x^2}{3x^2 + 1}$

(2) Find the points of inflection of the function

$$f(x) = \sin x + \cos x, \quad [0, 2\pi]$$

(3) Use the Second Derivative Test to find all relative extrema.

$$f(x) = \sqrt{x^2 + 1}$$