BC Calculus – Logistic Growth

The spread of a disease through a community can be modeled with the logistic equation $y = \frac{600}{1+59e^{-0.1t}}$, where y is the number of people infected after t days. How many people are infected when the disease is spreading the fastest?

(A) 10 (B) 59 (C) 60 (D) 300 (E) 600

The spread of a disease through a community can be modeled with the logistic equation $y = \frac{0.9}{1+45e^{-0.15t}}$, where y is the proportion of people infected after t days. According to the model, what percentage of people in the community will not become infected?

(A) 2% (B) 10% (C) 15% (D) 45% (E) 90%

The population $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt} = P\left(2 - \frac{P}{5000}\right)$, where the initial population is $P(0) = 3000$ and t is the time in years. What is $\lim_{t\to\infty} P(t)$?

(A) 2500 (B) 3000 (C) 4200 (D) 5000 (E) 10,000

Suppose a population of wolves grows according to the logistic differential equation $\frac{dP}{dt} = 3P - 0.01P^2$, where P is the number of wolves at time t, in years. Which of the following statements are true?

I. $\lim_{t\to\infty} P(t) = 300$

II. The growth rate of the wolf population is greatest when $P = 150$.

III. If $P > 300$, the population of wolves is increasing.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

Suppose the population of bears in a national park grows according to the logistic differential equation $\frac{dP}{dt} = 5P - 0.002P^2$, where P is the number of bears at time t in years.

(a) If $P(0) = 100$, then $\lim_{t\to\infty} P(t) =$ ____________. Sketch the graph of $P(t)$. For what values of P is the graph of P increasing? decreasing? Justify your answer.

(b) If $P(0) = 1500$, $\lim_{t \to \infty} P(t) =$ ____________. Sketch the graph of $P(t)$. For what values of P is the graph of P increasing? decreasing? Justify your answer.

(c) If $P(0) = 3000$, $\lim_{t \to \infty} P(t) =$ ____________. Sketch the graph of $P(t)$. For what values of P is the graph of P increasing? decreasing? Justify your answer.

(d) How many bears are in the park when the population of bears is growing the fastest? Justify your answer.

(Calculator Permitted) A population of animals is modeled by a function P that satisfies the logistic differential equation $\frac{dP}{dt} = 0.01P(100 - P)$, where t is measured in years.

(a) If $P(0) = 20$, solve for P as a function of t.

(b) Use your answer to (a) to find P when $t = 3$ years. Give exact and 3-decimal approximation.

(c) Use your answer to (a) to find t when $P = 80$ animals. Give exact and 3-decimal approximation.

(Calculator Permitted) The rate at which a rumor spreads through a high school of 2000 students can be modeled by the differential equation $\frac{dP}{dt} = 0.003P(2000 - P)$, where P is the number of students who have heard the rumor t hours after 9AM.

(a) How many students have heard the rumor when it is spreading the fastest?

(b) If $P(0) = 5$, solve for P as a function of t.

Suppose that a population develops according to the logistic equation $\frac{dP}{dt} = 0.05P - 0.0005P^2$ where t is measured in weeks.

(a) What is the carrying capacity/limit to growth?