WARM UP - Calculator Inactive

1. $\int x \tan^{-1} x \, dx$

2. Find *k* if $\int_0^9 \left(\frac{1}{\sqrt{x}} - k\right) dx = 12$

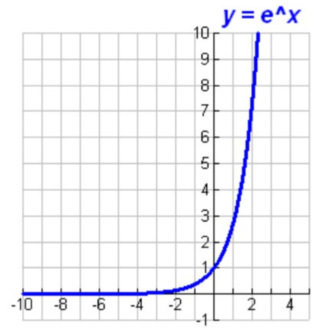
Logistic Growth

Objective:

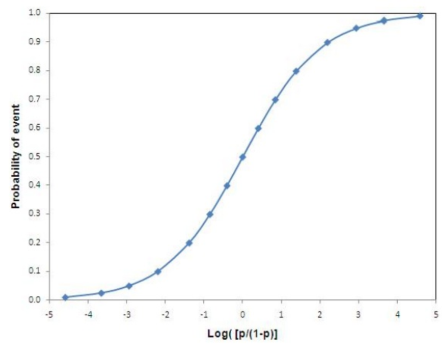
- Solve problems modeled by logistic growth.

Exponential vs. Logistic Growth

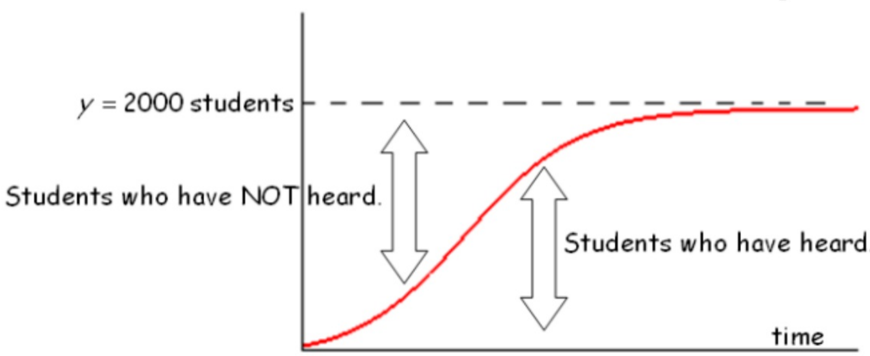
Exponential Growth -
grows at a growing rate
and is unrestricted.



Logistic Growth -
can start growing at an exponential rate, but the growth will slow. Restricted by horizontal asymptotes.

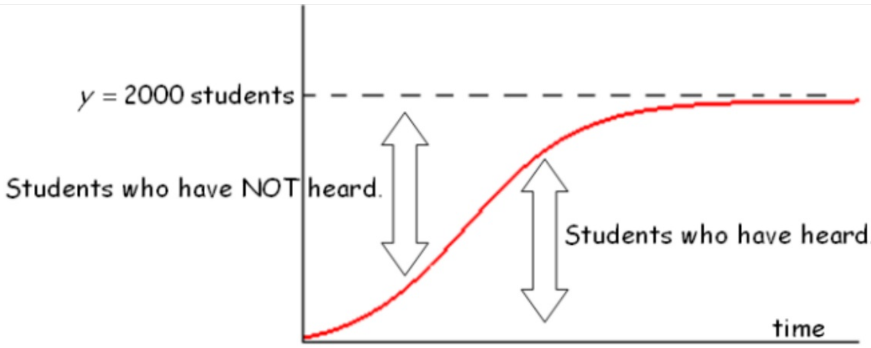


The rumor example



In this case, the growth rate is not only proportional to the current value, but also how far the current value is from the carrying capacity.

The rate at which the rumor spreads is directly proportional to BOTH the students who have heard the rumor AND the students who have yet to hear the rumor as the number of people hearing the rumor approaches 2000.



For quantities, y, that grow logistically with a carrying capacity of y = L, then....

$$\frac{dy}{dt} = ky(L - y)$$

If we solve the differential equation, we get:

$$\text{If } \frac{dy}{dt} = ky(L - y), \text{ then } y = \frac{L}{1 + Ce^{-Lkt}}$$

The population of Alaska from 1900 to 2000 can be modeled by the following logistic equation.

$$P(t) = \frac{895598}{1 + 71.57e^{-0.0516t}}$$

where P is the population and t years after 1900, with t=0 corresponding to 1900.

a. What is the predicted population in 2020?

b. How fast was the population changing in 1920? 1940? 1999?

c. When was Alaska growing the fastest, and what was the population then?

The growth rate of a population P of bears in a newly established wildlife preserve is modeled by the differential equation (where t measured in years)

$$\frac{dP}{dt} = 0.008P\left(100 - P\right)$$

a. What is the carrying capacity for bears in this wildlife preserve?

b. What is the population when the population is growing the fastest?

c. What is the rate of change of population when it is growing the fastest?

Suppose that a population develops according to the logistic differential equation $\frac{dP}{dt} = 0.2P - 0.002P^2$, where t is measured in weeks, $t \geq 0$.

a) If $P(0) = 5$, what is $\lim_{t \to \infty} P(t)$?

b) If $P(0) = 60$, what is $\lim_{t \to \infty} P(t)$?

c) If $P(0) = 120$, what is $\lim_{t \to \infty} P(t)$?

d) Sketch the solution curves for a), b), and c). Which one has an inflection point?

The rate at which the flu spreads through a community is modeled by the logistic differential equation $dP/dt = 3 - .001P$, where *t* is measured in days, $t \geq 0$.

(a) If $P(0)=50$, solve for P as a function of *t*.

(b) Use your solution to (a) to find the size of the population when $t = 2$ days.

(c) Use your solution from (a) to find the number of days that have occurred when the flu is spreading the fastest.

Quiz - Review Problems

1. $\int \frac{3x^3 + 4x + 11}{x^2 - 3x + 2}\,dx$

3. $\int_1^e \frac{\ln x}{5x}\,dx$

2. $\int x \tan^{-1}(x)\,dx$

4. $\int \frac{x^2 + 4}{3x^3 + 4x^2 - 4x}\,dx$