WARM UP

1. Find the solution of the differential equation:

$$\frac{dy}{dx} = xcos(x^2); \quad y(0) = \pi$$

2. The graph of the function $y = x^3 + 6x^2 + 7x - 2\cos x$ changes concavity at $x =$

(A) -1.58 (B) -1.63 (C) -1.67

(D) -1.89 (E) -2.33

Direct Comparison Test (DCT)
&
Limit Comparison Test (LCT)

Objective:

- Use the Direct Comparison Test and Limit Comparison Test to see if an infinite series converges or diverges.

Why Comparison Tests?

1. $\sum_{n=0}^{\infty} \frac{1}{2^n}$ is geometric, but $\sum_{n=0}^{\infty} \frac{n}{2^n}$ is not.
2. $\sum_{n=1}^{\infty} \frac{1}{n^3}$ is a p-series, but $\sum_{n=1}^{\infty} \frac{1}{n^3 + 1}$ is not.
3. $a_n = \frac{n}{(n^2 + 3)^2}$ is easily integrated, but $b_n = \frac{n^2}{(n^2 + 3)^2}$ is not.

Direct Comparison Test (DCT)

If $a_n \geq 0$ and $b_n \geq 0$,

1) If $\sum_{n=1}^{\infty} b_n$ converges and $0 \leq a_n \leq b_n$, then $\sum_{n=1}^{\infty} a_n$ _____.

2) If $\sum_{n=1}^{\infty} a_n$ diverges and $0 \leq a_n \leq b_n$, then $\sum_{n=1}^{\infty} b_n$ _____.

NOTE:You must state/show the inequality when stating the conclusion of the test!!

1. If the "larger" series converges, then the "smaller" series must also converge.
2. If the "smaller" series diverges, then the "larger" series must also diverge.

$$\sum_{n=1}^{\infty} \frac{1}{3n^2 + 2}$$

$$\sum_{n=0}^{\infty} \frac{4^n}{5^n + 3}$$

(e) $\sum_{n=1}^{\infty} \frac{\cos n}{2^n}$

$\sum_{n=0}^{\infty} e^{-n^2}$

Limit Comparison Test (LCT)

If $a_n \geq 0$ and $b_n \geq 0$, and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$ **or** $\lim_{n\to\infty} \frac{b_n}{a_n} = L$, where L is both finite and positive.

Then the two series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ either both converge or both diverge.

(a) $\sum_{n=1}^{\infty} \frac{1}{3n^2 - 4n + 5}$

(b) $\sum_{n=1}^{\infty} \frac{n^4 + 10}{4n^5 - n^3 + 7}$

$\sum_{n=1}^{\infty} \frac{2^n + 1}{5^n + 1}$

(d) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{3n-2}}$

Classwork: Determine if the following series converge or diverge. Use every test in this unit at least once.

23. $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{n}$

24. $\sum_{n=0}^{\infty} 5\left(-\frac{4}{3}\right)^n$

25. $\sum_{n=1}^{\infty} \frac{1}{5^n + 1}$

26. $\sum_{n=2}^{\infty} \frac{1}{n^3 - 8}$

27. $\sum_{n=1}^{\infty} \frac{2n}{3n - 2}$

28. $\sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+2}\right)$

29. $\sum_{n=1}^{\infty} \frac{n}{(n^2 + 1)^2}$

30. $\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$