Warm-Up

1. Evaluate: $\lim_{x \to -\infty} \frac{-3x}{\sqrt{4x^2 - 3}}$

2. Determine the vertical asymptotes of $y = \frac{2x^2 - 5x + 3}{2x^2 + 11x - 21}$

3. Find the value of c such that the limit exists at x = 1.

$$f(x) = \begin{cases} 2cx - 2, & x > 1 \\ 5, & x = 1 \\ x^2 + c, & x < 1 \end{cases}$$

Continuity and the Intermediate Value Theorem

Objective:

- Determine continuity at a point.
- Understand and use the Intermediate Value Theorem.

Definition of Continuity

Continuity at a Point

A function f is **continuous at c** when these three conditions are met.

1. $f(c)$ is defined.

2. $\lim_{x \to c} f(x)$ exists.

3. $\lim_{x \to c} f(x) = f(c)$

Continuity on an Open Interval

A function is **continuous on an open interval (a, b)** when the function is continuous at each point in the interval. A function that is continuous on the entire real number line $(-\infty, \infty)$ is **everywhere continuous.**

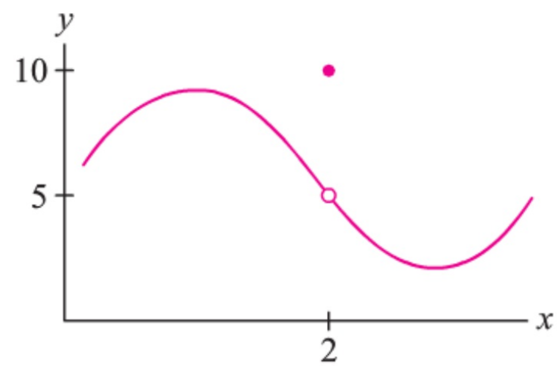
Ex.1:The function f(x) is defined as follows:

$$f(x) = \begin{cases} 4x - 11, x < 3 \\ kx^2, \quad x \geq 3 \end{cases}$$

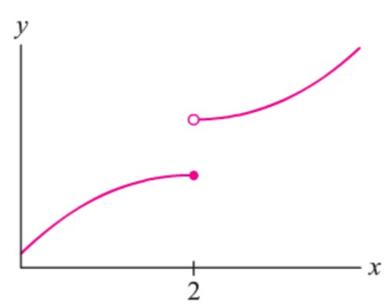
Find a value of *k* such that *f* is continuous for all *x*.

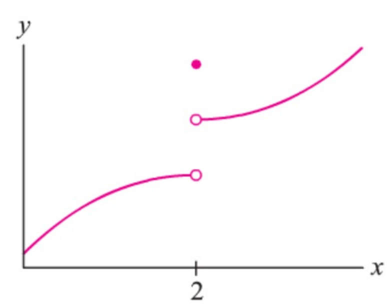
Types of Discontinuities

(1) Removable Discontinuity

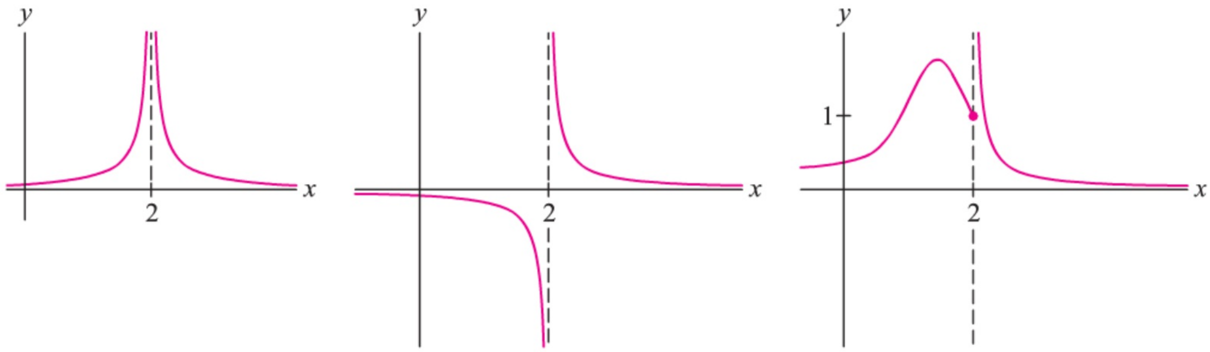


(2) Jump Discontinuity

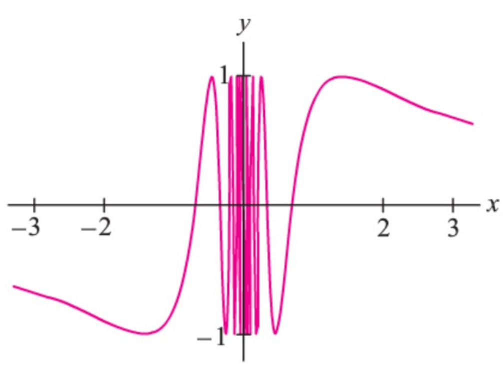




(3) Infinite Discontinuity



(4) Oscillating



Some of our most common functions are continuous every where on their domains (where the domains are defined)

- Polynomials
- Rational Functions
- Trig
- Root

Determine the points of discontinuity. State the type of discontinuity (removable, jump, infinite, or none of these).

Ex.2: $f(x) = \begin{cases} -2x + 7, x \neq 1 \\ 4, \quad x = 1 \end{cases}$

Ex.3: $h(x) = \begin{cases} \dfrac{x-2}{|x-2|}, x \neq 2 \\ -1, \quad x = 2 \end{cases}$

Intermediate Value Theorem

If *f* is <u>*continuous*</u> on the closed interval [a,b] and *w* is any number between f(a) and f(b), then there must be at least one number *c* in [a,b] such that f(c) = w.

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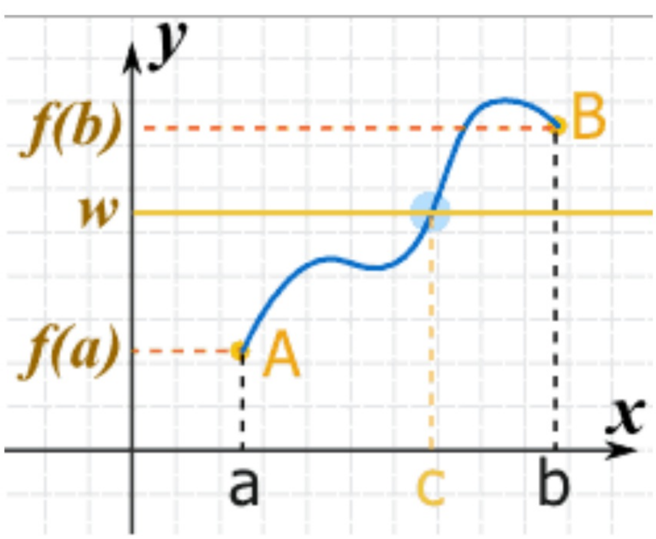

Ex.4: Use the Intermediate Value Theorem to prove that the polynomial function $f(x) = x^3 + 2x - 1$ has a zero in the interval [0,1].

Ex.5: Use the Intermediate Value Theorem to prove that the polynomial function $f(x) = x^2 - 4x + 2$ has some value of *c* such that $f(c) = 4$ in the interval [1,5].

Guided Practice

1. Which values for *k* make the function continuous at x = 0?

$$f(x) = \begin{cases} \ln(x + k), 0 < x < 3 \\ \cos(kx), \quad x \le 0 \end{cases}$$

2.

If $f(x) = \begin{cases} ax^2 - b, & x < -1 \\ 4, & x = -1 \\ 2ax + b, & x > -1 \end{cases}$, find the values of a and b such that $f(x)$ is continuous at $x = 1$.

3. The table below gives several measurement s of the velocity of a particle moving along a straight line. What is the smallest possible number of times where v(t) is exactly 4 meters/sec ($0 \le t \le 14$)?

t (sec)	0	5	7	9	14
v(t) (meter/sec)	3.35	4.25	2.75	2.55	4.70