

Rules for Differentiation

Objective:

- Find the derivative of functions using the product and quotient rule.
- Find second and higher order derivatives.

The Product Rule for Derivatives

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

or

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

$$\text{Ex. 1: } f(x) = (2x^5 - 7)(3x^3 + 14x)$$

The Quotient Rule for Derivatives

$$\left(\frac{f}{g}\right)'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

A trick to remember...

Call the numerator "high" and the denominator "low", then the derivative of the quotient is:

low D high - high D low
low low

Ex.2:

Given the function below, find its derivative.

$$y = \frac{\sqrt{x}}{1+x^2}$$

Higher derivatives are obtained by repeatedly differentiating a function.

<i>First derivative:</i>	y' ,	$f'(x)$,	$\frac{dy}{dx}$,	$\frac{d}{dx}[f(x)]$,
<i>Second derivative:</i>	y'' ,	$f''(x)$,	$\frac{d^2y}{dx^2}$,	$\frac{d^2}{dx^2}[f(x)]$,
<i>Third derivative:</i>	y''' ,	$f'''(x)$,	$\frac{d^3y}{dx^3}$,	$\frac{d^3}{dx^3}[f(x)]$,
<i>Fourth derivative:</i>	$y^{(4)}$,	$f^{(4)}(x)$,	$\frac{d^4y}{dx^4}$,	$\frac{d^4}{dx^4}[f(x)]$,
	\vdots			
<i>nth derivative:</i>	$y^{(n)}$,	$f^{(n)}(x)$,	$\frac{d^ny}{dx^n}$,	$\frac{d^n}{dx^n}[f(x)]$,

Ex.3: Find the $\frac{d^3y}{dx^3}$ for $y = 3x^4 - 2x^2 + 1$

Ex.4: Table problems

Use the chart to find $f'(3)$

$g(3)$	$g'(3)$	$h(3)$	$h'(3)$
4	-2	3	π

a) $f(x) = 4g(x) - \frac{1}{2}h(x) + 1$ b) $f(x) = g(x)h(x)$ c) $f(x) = \frac{g(x)}{2h(x)}$ d) $f(x) = \frac{g(x) - h(x)}{g(x)}$