WARM UP

1. Which of the following coefficient of x^4 in the Maclaurin polynomial generated by cos(3x)?

(a) 27/8 (b) 9 (c) 1/24 (d) 0 (e) -27/8

2. Determine if the following converges/diverges. Justify.

$$\sum_{n=1}^{\infty} \frac{(-1)^n 3^n}{n2^n}$$

Power Series: Day 1

Objective:

- Understand the definition of a power series.
- Find the radius and interval of convergence.
- Determine the endpoint of convergence of a power series

If x is a variable, then an infinite series of the form:

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \cdots a_n x^n + \cdots$$

is called a power series

$$\sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_1 (x-c) + a_2 (x-c)^2 + \cdots a_n (x-c)^n + \cdots$$

is a power series center at c, where c is a constant

A power series in x can be viewed as a function of x where the domain of f is the set of all x for which the power series converges.

Our job is to find what value of x the series converges.

For a power series centered at c, precisely one of the following is true:

(1) The series converges only at c (ALL power series converge at their centers!)

The radius is 0

The domain of f(x), also called the interval of convergence, would be a single point (the center).

(2) The series converges for all x (function and series are equal for all values)

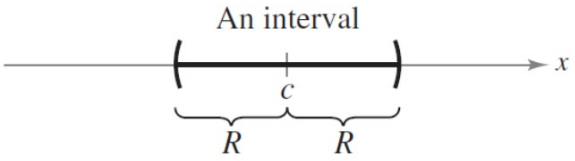
The radius is ∞

The IOC $(-\infty, \infty)$. The series converges absolutely for all x.

(3) There exist a $R > 0$ such that the series converges for $|x - c| < R$ and diverges for $|x - c| > R$

R is called the radius of convergence of the power series

IOC - $[(c - R, c + R)]$



You need to check each endpoint independently to see if the converges/diverges.

Ex.1: Find the nth term for the power series $f(x) = e^x$, centered at zero and then find the radius and interval of convergence for the representative power series.

Ex.2: Find the radius of convergence and the interval of convergence. Be sure to check the endpoints.

$$\sum_{n=0}^{\infty}\left(\frac{x}{3}\right)^n$$

Ex.3: Find the radius of convergence and the interval of convergence. Be sure to check the endpoints.

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(x-5)^n}{n2^n}$$

Ex.4: Find the radius of convergence and the interval of convergence. Be sure to check the endpoints.

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

Ex.5: Find the radius of convergence and the interval of convergence. Be sure to check the endpoints.

$$\sum_{n=0}^{\infty} n!(x-3)^n$$

CLASSWORK - Find the radius and interval of convergence of the power series.

$$\sum_{n=1}^{\infty} \frac{(4x)^n}{n^2} \qquad \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} \qquad \sum_{n=0}^{\infty} \left(\frac{x}{4}\right)^n \qquad \sum_{n=0}^{\infty} (2n)! \left(\frac{x}{3}\right)^n$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-1)^{n+1}}{n+1} \qquad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-4)^n}{n9^n}$$