Warm-Up

(1) If $f(2) = 4$, can you conclude anything about the limit of $f(x)$ as x approaches 2? Explain.

(2) Evaluate each limit.

$$\lim_{x\to\infty} \frac{-12x}{\sqrt{4x^2 - x}} \qquad \lim_{x\to-\infty} \frac{-12x}{\sqrt{4x^2 - x}}$$

(3) Evaluate each trigonometric expression.

$$\sin\frac{\pi}{3} = \qquad \tan\frac{3\pi}{2} =$$

Limits: Algebraic Approach

Objective:

- Use algebraic manipulation to evaluate limits.
- Evaluate limits of indeterminate forms $\left(\frac{0}{0}\right)$

Direct Substitution Method

Ex. 1: $\lim_{x \to 1} \frac{x^2 + x + 2}{x + 1}$

Ex. 2: $\lim_{x \to -5} 3$

Possible outcomes from Direct Substitution (rational functions):

(1) Defined Answers - like previous examples. Remember: 0/a = 0

(2) b/0 → the limit Does Not Exist
(∞, -∞, DNE)

(3) 0/0 → Indeterminate form.
Generally using factoring, simplifying fractions, or rationalization.

Ex. 3: $\lim_{x \to -3} \dfrac{x^2 + x - 6}{x + 3}$

Ex. 4: $\lim_{x \to 0} \dfrac{\dfrac{1}{x+4} - \dfrac{1}{4}}{x}$

Ex. 5: $\lim_{x \to 8} \dfrac{\sqrt{x-4}-2}{x-8}$

Ex. 6: $\lim_{x \to \pi/2} \dfrac{\tan x}{\sec x}$

Special Trigonometric Limits

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

$$\lim_{x\to 0}\frac{1-\cos x}{x}=0$$

Ex. 7: $\lim_{x\to 0}\frac{1-\cos\theta}{x}+\frac{4\sin x}{x}$

Ex. 8: $\lim_{x\to 0}\frac{\sin 3x}{x}$

Piecewise Functions:

Ex. 9: $\lim_{x \to 2} \begin{cases} x + 3, x > 2 \\ 3x - 3, x \leq 2 \end{cases}$

Find the constant a and b such that the limit exists at $x = -1$ and $x = 3$

$$f(x) = \begin{cases} 2, & x \leq -1 \\ ax + b, & -1 < x < 3 \\ -2, & x \geq 3 \end{cases}$$

Guided Practice

$$\lim_{x \to 0} \frac{\tan x}{x}$$

AP Exam Question

For what value of k does $\lim_{x \to 4} \frac{x^2 - x + k}{x - 4}$ exist?

AP Exam Question

(1998AB83) If $a \neq 0$, then $\lim_{x \to a} \frac{x^2 - a^2}{x^4 - a^4}$ is

(A) $\frac{1}{a^2}$ (B) $\frac{1}{2a^2}$ (C) $\frac{1}{6a^2}$ (D) 0 (E) nonexistent