WARM UP

1. The average rate of change of $f(x) = x^3$ over the interval $[a, b]$ is

A) $3b + 3a$

B) $b^2 + ab + a^2$

C) $\frac{b^2 + a^2}{2}$

D) $\frac{b^3 - a^3}{2}$

E) $\frac{b^4 - a^4}{4(b - a)}$

2. Find $\lim_{h \to 0} \frac{\tan\left(\frac{5\pi}{6} + h\right) - \tan\left(\frac{5\pi}{6}\right)}{h}$

Derivatives of Inverses

Objective:

- Find the derivatives of inverse functions.

Often, we name an inverse function $f^{-1}(x)$ as $g(x)$. If we then let $f^{-1}(x) = g(x)$, then the domain of f is the range of g, and the domain of g is the range of f.

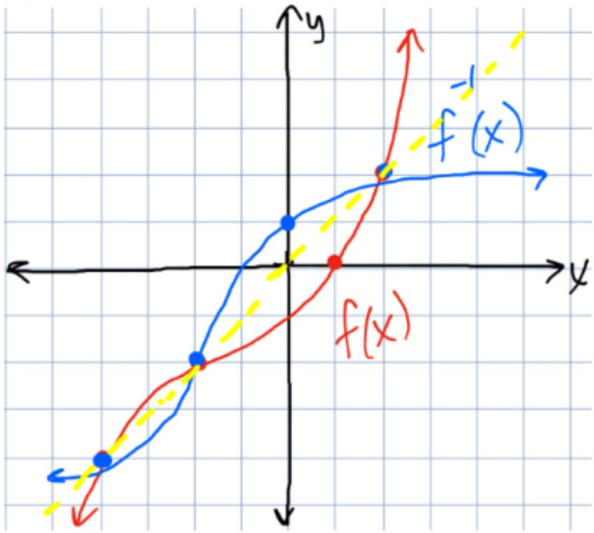
$f(a) = b$ means $g(b) = a$. The ordered pairs $f:(a,b)$ and $g:(b,a)$ are corresponding points.

A function $g(x)$ is said to be the inverse function of the function $f(x)$ if:

$f(g(x)) = x = g(f(x))$ for all x in the domain of f and g.

Graphical consequence of Inverse Functions:

The graphs of a function $f(x)$ and its inverse $f^{-1}(x)$ are reflections of each other across the line $y = x$.



Inverse functions, at corresponding points, have reciprocal slopes.

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Let f and g be inverse functions, such that $f(g(x)) = x = g(f(x))$ with $f(a) = b$ and $g(b) = a$, then

$$g'(b) = \frac{1}{f'(a)}$$

Ex. 1: Given $f(x) = 2x^3 + 1$, find:

a) f(1)

b) f '(1)

c) $(f^{-1})'(3)$

Ex.2:

If $f(x) = x^5 + 2x - 1$

(a) Find f(1)

(b) Find $(f^{-1})'(2)$

Ex.3:

If $m(n(x)) = x = n(m(n))$ and m(5) = -9 and m'(5) = -8/11, then find n'(-9)

Ex.4: Let *f* be a differentiable function with the following values.

x	$f(x)$	$f'(x)$
3	15	-8
6	3	-2

The function *g* is a differentiable and $g(x) = f^{-1}(x)$ for all x. What is the value $g'(3)$?

Ex.5:Selected values of differentiable functions *f, g*, and *f* ' are given below. If *f(g(x)) = x = g(f(x))*, find *g '(-3)*.

x	$f(x)$	$f'(x)$	$g(x)$
–3	4	6	7
4	–8	–2	–3
7	–3	2	9