WARM UP

1. $\int_{e}^{\infty} \frac{1}{x(\ln x)^2}\,dx$

2.

The sum of the infinite geometric series $\frac{3}{2}+\frac{9}{16}+\frac{27}{128}+\frac{81}{1,024}+\dots$ is

(A) 1.60 (B) 2.35 (C) 2.40

(D) 2.45 (E) 2.50

Integral Test and p-Series Test

Objective:

- Use the Integral Test and p-Series Test to determine if an infinite series converges.
- Use the integral test to approximate the sum of the series.

THEOREM 9.10 The Integral Test

If f is positive, continuous, and decreasing for $x \geq 1$ and $a_n = f(n)$, then

$$\sum_{n=1}^{\infty} a_n \quad \text{and} \quad \int_1^{\infty} f(x)\,dx$$

either both converge or both diverge.

Note:

- This does NOT mean that the series converges to the value of the integral
- The function need only be decreasing for all $x > k$ for some $k \geq 1$
- We can find an interval in which the sum resides using a partial sum of the series and the remainder R_n

$$0 \leq R_n \leq \int_n^{\infty} f(x)dx$$

Determine if the following series converges or diverges. If it converges, find an interval in which the sum resides using S_4

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

Determine if the following series converges or diverges. If it converges, find an interval in which the sum resides using S_4

(b) $\sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$

Prove the series converges. Approximate the sum by using the first six terms.

$$\sum_{n=1}^{\infty} \frac{1}{n^4}$$

A series in the form of:

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdot \cdot \cdot$$

p-series

is a *p*-series, where *p* is a positive constant.

If $p = 1$, the series is called the Harmonic series.

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \cdot \cdot \cdot$$

Harmonic series

THEOREM 9.11 Convergence of *p*-Series

The *p*-series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \cdot \cdot \cdot$$

converges for $p > 1$, and diverges for $0 < p \leq 1$.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$$

$$1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \frac{1}{5\sqrt{5}}$$

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n}$$

$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{n}}$$

Practice - determine if the following converge or diverge. Find the exact sum if possible, or approximation using S_5.

$$\sum_{n=0}^{\infty} \left(\frac{2}{7}\right)^n$$

$$\sum_{n=1}^{\infty} \frac{1+3n^2+n^3}{4n^3-5n+2}$$

$$\sum_{n=1}^{\infty} \frac{n^n}{n!}$$

$$\sum_{n=1}^{\infty} \frac{4}{n^3}$$

$$\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}+\cdots$$

$$\sum_{n=1}^{\infty} \frac{3n}{2n^2+3}$$

$$2+\frac{1}{2}+\frac{1}{8}+\frac{1}{32}+\cdots$$