WARM UP

1. Find the absolute extrema on the function on the closed interval.

$$f(x) = \frac{x}{\sqrt{x^2 + 1}}, \quad [0, 2]$$

2. Determine if the MVT can be applied to *f* on the closed interval. If so, find all values of *c* guaranteed by the MVT.

$$f(x) = x^{2/3}, \quad [1, 8]$$

First Derivative Test for Local Extrema

Objective:

- Determine intervals on which a function is increasing or decreasing.
- Apply the First Derivative Test to find relative extrema.

Increasing and Decreasing

THEOREM 3.5 Test for Increasing and Decreasing Functions

Let f be a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b).

1. If $f'(x) > 0$ for all x in (a, b), then f is increasing on $[a, b]$.
2. If $f'(x) < 0$ for all x in (a, b), then f is decreasing on $[a, b]$.
3. If $f'(x) = 0$ for all x in (a, b), then f is constant on $[a, b]$.

First Derivative Test for Local Extrema

The following test applies to a continuous function f(x).

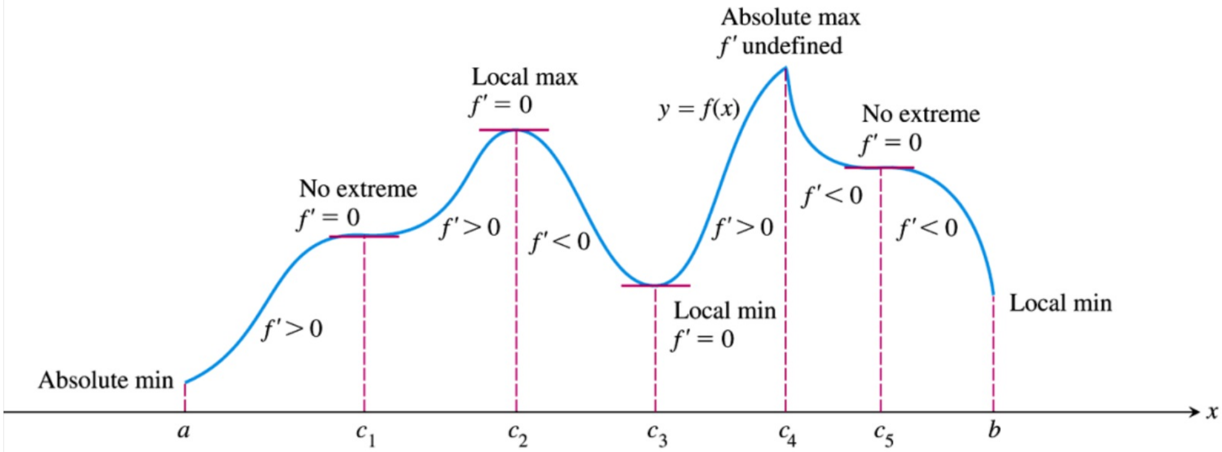
At a critical point c **:**

1. If f' changes sign from positive to negative at c, then f has a local maximum value at c.
2. If f' changes sign from negative to positive at c, then f has a local minimum value at c.
3. If f' does not change sign at c, then f has no local extreme value at c.

Important Idea

If $f(x)$ is a **continuous function**, then $f'(x)$ can only change its sign at a **critical value**.

If $f(x)$ is a **discontinuous function**, then $f'(x)$ can change its sign either at a **critical value** or a **discontinuity.**



Ex.1: Find the open intervals on which f(x) is increasing and/or decreasing. Justify

$$f(x) = x^3 - \frac{3}{2}x^2$$

Ex. 2: For f(x) = $3x^4 - 4x^3 - 12x^2 + 5$

A. Find the open intervals on which f(x) is increasing and/or decreasing. Justify.

B. Determine the x-values of any local max/mins of f(x). Justify.

Ex. 3: Find the values of any relative extrema of the function. Justify.

$$f(x) = \frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{7}{2}x^2 + 4x - 4$$

Ex. 4: Find the values of any relative extrema of the function f(x) = 1/2x - sinx on the interval [0,2π]. Justify

Ex. 4: Find the relative extrema of $f(x) = \left(x^2 - 4\right)^{2/3}$ Justify.

Applying the First Derivative Test In Exercises 17–40, (a) find the critical numbers of f (if any), (b) find the open interval(s) on which the function is increasing or decreasing, (c) apply the First Derivative Test to identify all relative extrema, and (d) use a graphing utility to confirm your results.

$$f(x) = (x + 2)^{2/3}$$

$$f(x) = (x - 1)^2(x + 3)$$

$$f(x) = x + 2\sin x \qquad (0, 2\pi)$$

TICKET OUT

Find the x-coordiantes of the relative extrema of the T(k). Justify.

$$T(k) = \sqrt[3]{k^2}\,(2k - 1)$$