Exponential Growth & Decay

Quantities that grow/decrease by a factor or a percentage at regular intervals, are exponential.

The rate of change of a quantity is directly proportional to that quantity itself.

$$\frac{dy}{dt} = ky$$

growth ($k > 0$) or decay ($k < 0$)

Solve the differential equation $\frac{dy}{dt} = ky$

Memorize

If $\frac{dy}{dt} = ky$, then $y = Ce^{kt}$, where C is the initial amount present (y-intercept)

Ex.1:The population of a bacteria in culture increased from 400 to 1600 in three hours. Assuming that the bacteria population, P, grows according to the rate,where t is the time in hours.

$$\frac{dP}{dt} = kP$$

a) Find the value of k.

b) How fast is the population increasing when the population is 3000?

Ex.2: Radium-226 loses its mass at a rate directly proportional to its mass. If its half-life is 1590 years, and if we start with a sample of radium-226 with mass of 100mg,

a) Find the formula for the mass, M(t) that remains after t years.

b) How many mg of the original sample remains after 1000 years?

Ex.3: Bacteria in a certain culture increase at a rate proportional to the number present. If the number of bacteria doubles in three hours, in how many hours will the number of bacteria triple?

(A) $\frac{3\ln 3}{\ln 2}$ (B) $\frac{2\ln 3}{\ln 2}$ (C) $\frac{\ln 3}{\ln 2}$ (D) $\ln\left(\frac{27}{2}\right)$ (E) $\ln\left(\frac{9}{2}\right)$

Ex.4:

In which of the following models is $\frac{dy}{dt}$ directly proportional to y?

I. $y = e^{kt} + C$

II. $y = Ce^{kt}$

III. $y = 28^{kt}$

IV. $y = 3\left(\frac{1}{2}\right)^{3t+1}$

(A) I only (B) II only (C) I and II only (D) II and III only (E) II, III, and IV (F) all of them