

WARM UP

1.
$$\int \frac{3x^2 + x - 1}{x^2} dx$$

2.

The data for the acceleration $a(t)$ of a car from 0 to 15 seconds are given in the table below. If the velocity at $t = 0$ is 5 ft/sec, which of the following gives the approximate velocity at $t = 15$ using a Trapezoidal sum?

t (sec)	0	3	6	9	12	15
$a(t)$ (ft/sec ²)	4	8	6	9	10	10

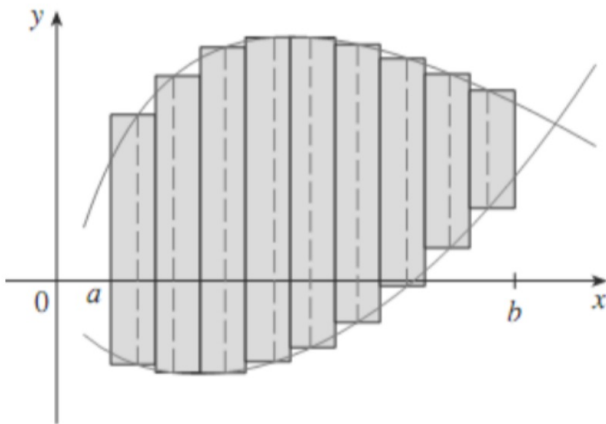
(A) 47 ft/sec (B) 52 ft/sec (C) 120 ft/sec (D) 125 ft/sec (E) 141 ft/sec

AREA BETWEEN TWO CURVES

Objective:

- Find the area between curves

The height of each rectangle on this interval can be represented by the function $h(x)$, where $h(x) = f(x) - g(x)$ or $h(x) = \text{TOP} - \text{BOTTOM}$



$$A = \int_a^b [f(x) - g(x)] dx$$

$$\text{Area} = \int_a^b (\text{upper curve} - \text{lower curve}) dx$$

STEPS:

- Sketch the graphs.
- Identify the enclosed or bounded region.
- Decide how to slice the region - vertically or horizontally.
- Find intervals of integration.
- Set up the equation for the height of the rectangles and integrate.

Note: Unlike the area under a curve, the area between two curves is always positive.

Ex.1 (No Calc): Find the area between the curves $y = x^2$ and $y = 2x - x^2$.

Ex.2 (No calc): Determine the area of the region bounded by $y = -x^2 + 3x$ and $y = 2x^3 - x^2 - 5x$. Set up, but do not integrate.

Ex.3 (Calc): Find the area between the curves
 $y = x^4 - x$ and $y = \frac{x}{\sqrt{x^2 + 1}}$

Ex.4 (Calc): Determine the area of the region
bounded by $y = x+1$, $x=2$, and $y = xe^{-x^2}$

Ex.5 (No Calc): Let R be a region in the first quadrant under the graph of $y = \frac{1}{\sqrt{x}}$ for $4 \leq x \leq 9$. If the line $x = k$ divides the region into two equal areas, what is the value of k ?

CLASSWORK: Find the area bounded by the graphs. Show all work

1. $y = x^2 + x + 2$, $y = -x$, $x = 0$, and $x = 1$

2. $y = 6 - x^2$ and $y = x$

3. $x = 0$, and the first intersection $y = \sin x$ and $y = \cos x$

4. $2x^3 - x^2 - 14x$ and $y = 4x - x^2$

5. $f(x) = 2^x$ and $g(x) = (3/2)x + 1$ (calc)

6. $f(x) = 6x/(x^2+1)$, $y = 0$, $0 \leq x \leq 3$ (calc)