Taylor Polynomial Practice

(1) $e^{2x} = 1 + 2x + \frac{4x^2}{2!} + \frac{8x^3}{3!} + \frac{16x^4}{4!} + \ldots + \frac{(2x)^n}{n!} + \ldots$

(2) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} + \ldots + \frac{(-1)^n x^{2n}}{(2n)!} + \ldots$

(3) $xe^{2x} = x + 2x^2 + \frac{4x^3}{2!} + \frac{8x^4}{3!} + \ldots + \frac{2^n x^{n+1}}{n!} + \ldots$

(4) $f(x) = \frac{1}{x}$ $\quad f(1) = 1$

$f'(x) = -x^{-2}$ $\quad f'(1) = -1$

$f''(x) = 2x^{-3}$ $\quad f''(1) = 2$

$f'''(x) = -6x^{-4}$ $\quad f'''(1) = -6$

$f^{(4)}(x) = 24x^{-5}$ $\quad f^{(4)}(1) = 24$

$f^{(5)}(x) = -120x^{-6}$ $\quad f^{(5)}(1) = -120$

$\frac{1}{x} = 1 - (x-1) + \frac{2(x-1)^2}{2!} - \frac{6(x-1)^3}{3!} + \frac{24(x-1)^4}{4!} - \frac{120(x-1)^5}{5!} + \ldots$

$\frac{1}{x} = 1 - (x-1) + (x-1)^2 - (x-1)^3 + (x-1)^4 - (x-1)^5 + \ldots + (x-1)^n + \ldots$

(5) $\ln x = (x-1) - \frac{(x-1)^2}{2!} + \frac{2(x-1)^3}{3!} - \frac{6(x-1)^4}{4!} + \frac{24(x-1)^5}{5!} + \ldots$

$\ln x = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \frac{(x-1)^5}{5} + \ldots + \frac{(x-1)^n}{n} + \ldots$

(6) $f(\frac{1}{2}) \approx P_4(\frac{1}{2}) = 2.7083$

(7) (a) $f(0) = 0$ (b) $f'(0) = 3$ (c) $\frac{f''(0)}{2!} = 0$ $\quad f''(0) = 0$

(d) $\frac{f'''(0)}{3!} = 4$ $\quad f'''(0) = 24$

(8) (a) $P_3(x) = 3 - 2(x-5) + \frac{(x-5)^2}{2!} - \frac{3(x-5)^3}{3!}$

(b) $g(4.9) \approx P_3(4.9) = 3.2055$

(9) $a > 0$

$b < 0$

$c < 0$

(10) $a < 0$

$b < 0$

$c > 0$

(11) $\int_0^1 \frac{\sin t}{t}\,dt \approx \int_0^1 \frac{t - \frac{t^3}{3!} + \frac{t^5}{5!}}{t}\,dt = \int_0^1 \left[1 - \frac{t^2}{6} + \frac{t^4}{120}\right]dt = \left[t - \frac{t^3}{18} + \frac{t^5}{600}\right]_0^1 =$

$\frac{1703}{1800} \approx 0.9461$