WARM UP

1. The table shows the velocity of a rocket at different time values. Using correct units, explain the meaning of $\int_{10}^{70} v(t)\,dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\,dt$

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

2 $\frac{1}{2}\int e^{\frac{t}{2}}\,dt =$

(D) $2e^{\frac{t}{2}} + C$

(E) $e^{t} + C$

(A) $e^{-t} + C$

(B) $e^{-\frac{t}{2}} + C$

(C) $e^{\frac{t}{2}} + C$

Parametric Equations (Day 2)

Objective:

- Find the velocity and acceleration vector when given a position vector.
- Find the displacement and distance traveled for a given interval of time.

$x'(t) = \frac{dx}{dt}$ is the rate at which the x-coordinate is changing with respect to t or the velocity of the particle in the horizontal direction.

$y'(t) = \frac{dy}{dt}$ is the rate at which the y-coordinate is changing with respect to t or the velocity of the particle in the vertical direction.

$\vec{s} = \langle x(t), y(t) \rangle = (x(t), y(t))$ is the position at any time t.

$\vec{v} = \langle x'(t), y'(t) \rangle = (x'(t), y'(t))$ is the velocity vector at any time t.

$\vec{a} = \langle x''(t), y''(t) \rangle = (x''(t), y''(t))$ is the acceleration vector at any time t.

*note: the vectors may be contained within
< > or ( ).

Ex 1: A particle moves in the xy-plane so that at any time t, the position of the particle is given by:

$x(t) = t^3 + 4t^2$ and $y(t) = t^4 - t^3$.

A. Find the velocity vector when t = 1.

B. Find the acceleration vector when t=2.

$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ is the **speed of the particle** or the **magnitude (length) of the velocity vector.**

$\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ is the **length of the arc (or arc length) of the curve from** $t = a$ to $t = b$ or the **distance traveled by the particle from** $t = a$ to $t = b$.

Displacement & Distance Traveled

Suppose a particle moves along a path in the plane so that its velocity at any time t is $\vec{v}(t) = (x'(t), y'(t))$, then the **displacement** from $t = a$ to $t = b$ is given by the vector

$$\left\langle \int_a^b x'(t)\,dt, \int_a^b y'(t)\,dt \right\rangle.$$

The preceding vector is added to the position at time $t = a$ to get the **position** at time $t = b$.

The **distance traveled** from $t = a$ to $t = b$ is the arc length

$$\int_a^b |\vec{v}(t)|\,dt = \int_a^b \sqrt{x'(t)^2 + y'(t)^2}\,dt.$$

Example (calculator):

An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with $\frac{dx}{dt} = \sin(t^3)$, $\frac{dy}{dt} = \cos(t^2)$. At time $t = 2$, the object is at the position $(1, 4)$.

(a) Find the acceleration vector for the particle at $t = 2$.

(b) Write the equation of the tangent line to the curve at the point where $t = 2$.

(c) Find the speed of the vector at $t = 2$.

(d) Find the position of the particle at time $t = 1$.

Question 3

An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \geq 0$ with $\frac{dx}{dt} = 3 + \cos\left(t^2\right)$. The derivative $\frac{dy}{dt}$ is not explicitly given. At time $t = 2$, the object is at position $(1, 8)$.

(a) Find the x-coordinate of the position of the object at time $t = 4$.

(b) At time $t = 2$, the value of $\frac{dy}{dt}$ is -7. Write an equation for the line tangent to the curve at the point $(x(2), y(2))$.

(c) Find the speed of the object at time $t = 2$.

(d) For $t \geq 3$, the line tangent to the curve at $(x(t), y(t))$ has a slope of $2t + 1$. Find the acceleration vector of the object at time $t = 4$.

(a)
$$x(4) = x(2) + \int_2^4 \left(3 + \cos\left(t^2\right)\right) dt$$
$$= 1 + \int_2^4 \left(3 + \cos\left(t^2\right)\right) dt = 7.132 \text{ or } 7.133$$

$$3 : \begin{cases} 1 : \int_2^4 \left(3 + \cos\left(t^2\right)\right) dt \\ 1 : \text{handles initial condition} \\ 1 : \text{answer} \end{cases}$$

(b)
$$\left.\frac{dy}{dx}\right|_{t=2} = \left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right|_{t=2} = \frac{-7}{3 + \cos 4} = -2.983$$
$$y - 8 = -2.983(x - 1)$$

$$2 : \begin{cases} 1 : \text{finds } \left.\frac{dy}{dx}\right|_{t=2} \\ 1 : \text{equation} \end{cases}$$

(c) The speed of the object at time $t = 2$ is
$\sqrt{(x'(2))^2 + (y'(2))^2} = 7.382$ or 7.383.

1 : answer

(d) $x''(4) = 2.303$

$y'(t) = \frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = (2t+1)\left(3 + \cos\left(t^2\right)\right)$

$y''(4) = 24.813$ or 24.814

The acceleration vector at $t = 4$ is
$\langle 2.303, 24.813 \rangle$ or $\langle 2.303, 24.814 \rangle$.

$$3 : \begin{cases} 1 : x''(4) \\ 1 : \frac{dy}{dt} \\ 1 : \text{answer} \end{cases}$$