WARM UP - Calculator Active

1. A particle moves along a straight line with velocity given by $v(t) = 7 - 1.01^{-t^2}$ at t>0. What is the acceleration of the particle at time t = 3?

(a) -0.914 (b) 0.055 (c) 5.486
(d) 6.086 (e) 18.087

The function f has first derivative given by $f'(x) = \frac{\sqrt{x}}{1+x+x^3}$. What is the x-coordinate of the inflection point of the graph of f?

A) 1.008 B) 0.473 C) 0 D) –0.278 E) The graph of f has no inflection point

Straight Line Motion
(Revisited)
Integration as Net Change

Objective:

- Use integration to solve problems involving particle motion.
- Find the total distance a particle travels.

Position
$s(t)$

$$s(t) = \int v(t)dt$$

Velocity
$v(t) = s'(t)$

$$v(t) = \int a(t)dt$$

Acceleration
$a(t) = v'(t) = s''(t)$

$$\text{Position (at } t = b) = s(b) = s(a) + \int_a^b v(t)dt$$

Distance vs Displacement

To find the displacement (position shift) from the velocity, just integrate the velocity. The negative areas below the x-axis subtracts from the total displacement.

$$\text{Displacement} = \int_a^b V(t)\,dt$$

To find the distanced traveled use the absolute value.

$$\text{Distance Traveled} = \int_a^b \left|V(t)\right|\,dt$$

Find where the velocity function changes direction and integrate in pieces (take abs.value of each)

Example 1: 2003 Form B #4 Calc. Inactive

A particle moves along the x-axis with a velocity at time $t \geq 0$ given by: $v(t) = -1 + e^{1-t}$.

(a) Find the acceleration of the particle at time t = 3.

(b) Is the speed of the particle increasing at time t = 3? Give a reason for your answer.

$$v(t) = -1 + e^{1-t}.$$

(c) Find all values of t at which the particle changes direction. Justify your answer.

(d) Find the total distance traveled by the particle over the time interval [0,3].

Example 2: The graph below shows several velocities at several different times of a particle moving along the x-axis. Its position at t = 4 is x = 10. Using midpoint Riemann sums using two intervals, estimate the position t = 16.

time (secs)	4	6	8	12	16
velocity (ft/sec)	5	6	8	10	14

Example 3: Calc Active

A particle moves along the x-axis so that at any time $t \geq 0$ its velocity is given by $v(t) = \cos(2 - t^2)$. The position of the particle is 3 at time $t = 0$. What is the position of the particle when its velocity is first equal to 0?

Example 4: Calc Active

A particle moves along the x-axis so that at any time $t \geq 0$, its acceleration is given by $a(t) = \ln(1 + 2^t)$. If the velocity of the particle is 2 at time $t = 1$, then the velocity of the particle at $t = 0$ is

Classwork:

AP® CALCULUS AB
2004 SCORING GUIDELINES (Form B)

Question 3

A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of $v(t)$ for $0 \le t \le 40$ are shown in the table above.

t (min)	0	5	10	15	20	25	30	35	40
$v(t)$ (mpm)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

(a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t)\,dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t)\,dt$ in terms of the plane's flight.

(b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.

(c) The function f, defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \le t \le 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicates units of measure.

(d) According to the model f, given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval $0 \le t \le 40$?