Warm-up

1. Identify any holes in the graph: $f(x) = \frac{x^2 - x - 6}{x - 3}$

2. Factor: $4x^2 + 13xy - 12y^2$

3. True or False? If it is false, explain why or give an example that shows it is false.

If *f (x)* is undefined at $x = c$, then the limit of *f(x)* as *x* approaches *c* does not exist.

Limits to Infinity

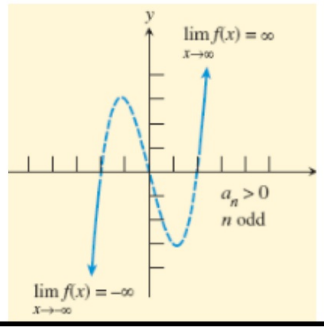
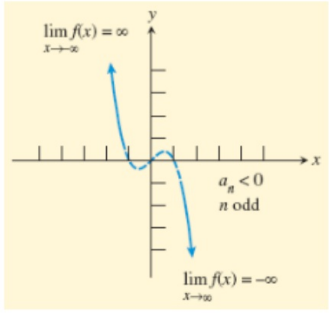
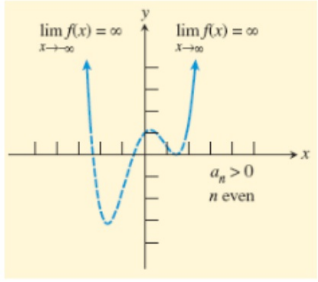
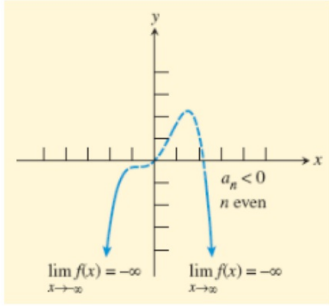
Objective:

- Describe **end-behaviors** of functions using limit notation (limits to infinity)

End Behavior for Polynomials - Limits as Infinity

Leading Term Test for End Behavior

$x \to \pm\infty$

	leading coefficient is positive	leading coefficient is negative
odd degree		
even degree		

Examples:

1. $\lim_{x\to\infty} 2x^3 - 4x^2 + 3x - 1$

2. $\lim_{x\to-\infty} -7x^2 + 2x + 10$

3. $\lim_{x\to\infty} -6x^3 + 5x^2 - 2x + 2$

FACT:

1. If *r* is a positive rational number and *c* is any real number then, $\lim_{x\to\infty} \frac{c}{x^r} = 0$

2. If *r* is a positive rational number, *c* is any real number and x^r is defined for $x < 0$ then, $\lim_{x\to-\infty} \frac{c}{x^r} = 0$

Example 4: $\lim_{x\to\infty} \frac{2x^3 - 4x + 7}{6x + 4x^3 + x^4}$

Short cuts for rational functions as limits $x \to \pm\infty$

f(x) = $\frac{\text{smaller degree}}{\text{larger degree}}$ limit = 0

f(x) = $\frac{\text{same degree}}{\text{same degree}}$ limit = $\frac{\text{leading coefficient}}{\text{leading coefficient}}$

f(x) = $\frac{\text{larger degree}}{\text{smaller degree}}$ limit = $\pm\infty$

(do previous method to check)

Examples:

5. $\lim_{x\to\infty} \frac{1-3x^2}{4x^2-3x+5}$

6. $\lim_{x\to\infty} \frac{x^2-2x+6}{2x^3-3}$

7. $\lim_{x\to-\infty} \frac{4x^2-2x+1}{2x-3}$

Example 8:

$$\lim_{x\to\infty} \frac{3^x - 1}{3^x + 5}$$

$$\lim_{x\to-\infty} \frac{3^x - 1}{3^x + 5}$$

Example 9:

$$\lim_{x\to\infty} \frac{2x}{\sqrt{x^2 - 3}}$$

$$\lim_{x\to-\infty} \frac{2x}{\sqrt{x^2 - 3}}$$