WARM UP

1. $P_3(x) = 0 + 1x + \frac{2}{3!}x^3 = x + \frac{2}{3}x^3$

If $f(0) = 0$, $f'(0) = 1$, $f''(0) = 0$, and $f'''(0) = 2$, then which of the following is the third-order Taylor polynomial generated by $f(x)$ at $x = 0$?

(A) $2x^3 + x$ (B) $\frac{1}{3}x^3 + \frac{1}{2}x$ (C) $\frac{2}{3}x^3 + x$ (D) $2x^3 - x$ $\boxed{\text{(E) } \frac{1}{3}x^3 + x}$

2. If a function function f is approximated by the 3rd degree Taylor Polynomial centered at x = 2, what is f '''(2)?

$P_3(x) = 4 - 3(x - 2) + 2(x - 2)^2 - 7(x - 2)^3$

$\frac{f^3(x)}{3!} = -7$ $\boxed{f^3(x) = -42}$

Intro intoTaylor Series

Objective:

- Generate the *n*th term (general term) for e^x, sinx, and cosx.
- Create Maclaurin Series for functions by modifiying e^x, sinx, and cosx.

Maclaurin series to memorize. Find the general term (nth term) and also write in sigma notation.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots \boxed{+\frac{x^n}{n!}+\cdots}$$

$$\boxed{\sum_{n=0}^{\infty} \frac{x^n}{n!}}$$

* Starting at center 0

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \boxed{+\frac{(-1)^n x^{2n+1}}{(2n+1)!}+\cdots}$$

$$\boxed{\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \boxed{+\frac{(-1)^n x^{2n}}{(2n)!}+\cdots}$$

$$\boxed{\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}}$$

You can manipulate these Maclaurin series to create new series.

1. Substitute into a series for x
2. Multiply or divide the series by a constant and/or a variable.
3. Add or subtract two series.

***Only works when centered at <u>zero</u>. If not, you have to generate the ~~polynomial~~ series using <u>Taylor's</u> rule.**

Write the first four non-zero terms of the Maclaurin ~~Polynomial~~ Series and the general term for the following:

(a) g(x) = sin(2x)

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \dots$$

$$\sin(2x) = (2x) - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} - \frac{(2x)^7}{7!} + \dots + \frac{(-1)^n (2x)^{2n+1}}{(2n+1)!} + \dots$$

(b) f(x) = xcos(3x)

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots + \frac{(-1)^n x^{2n}}{(2n)!} + \dots$$

$$\cos(3x) = 1 - \frac{(3x)^2}{2!} + \frac{(3x)^4}{4!} - \frac{(3x)^6}{6!} + \dots + \frac{(-1)^n (3x)^{2n}}{(2n)!} + \dots$$

$$x\cos(3x) = x - \frac{x(3x)^2}{2!} + \frac{x(3x)^4}{4!} - \frac{x(3x)^6}{6!} + \dots + \frac{(-1)^n x(3x)^{2n}}{(2n)!} + \dots$$

$$x\cos(3x) = x - \frac{3^2 x^3}{2!} + \frac{3^4 x^5}{4!} - \frac{3^6 x^7}{6!} + \dots + \frac{(-1)^n 3^{2n} x^{2n+1}}{(2n)!}$$

(c) $f(x) = 4e^{x^2}$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots$$

$$e^{x^2} = 1 + (x^2) + \frac{(x^2)^2}{2!} + \frac{(x^2)^3}{3!} + \dots + \frac{(x^2)^n}{n!} + \dots$$

$$4e^{x^2} = 4 + 4x^2 + \frac{4x^4}{2!} + \frac{4x^6}{3!} + \dots + \frac{4x^{2n}}{n!} + \dots$$