

WARM UP

1.

If $f(0) = 0$, $f'(0) = 1$, $f''(0) = 0$, and $f'''(0) = 2$, then which of the following is the third-order Taylor polynomial generated by $f(x)$ at $x = 0$?

(A) $2x^3 + x$ (B) $\frac{1}{3}x^3 + \frac{1}{2}x$ (C) $\frac{2}{3}x^3 + x$ (D) $2x^3 - x$ (E) $\frac{1}{3}x^3 + x$

2. If a function f is approximated by the 3rd degree Taylor Polynomial centered at $x = 2$, what is $f'''(2)$?

$$P_3(x) = 4 - 3(x - 2) + 2(x - 2)^2 - 7(x - 2)^3$$

Intro into Taylor Series

Objective:

- Generate the n th term (general term) for e^x , $\sin x$, and $\cos x$.
- Create Maclaurin Series for functions by modifying e^x , $\sin x$, and $\cos x$.

Maclaurin series to memorize. Find the general term (nth term) and also write in sigma notation.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

You can manipulate these Maclaurin series to create new series.

1. Substitute into a series for x
2. Multiply or divide the series by a constant and/or a variable.
3. Add or subtract two series.

***Only works when centered at zero. If not, you have to generate the polynomial using Taylor's rule.**

Write the first four non-zero terms of the Maclaurin Polynomial and the general term for the following:

(a) $g(x) = \sin(2x)$

$$(b) f(x) = x\cos(3x)$$

$$(c) f(x) = 4e^{x^2}$$