WARM UP - No Calculator

1. $\int \frac{1}{x^2 - 9} dx$

2. If the region enclosed by the y-axis, the line y = 2, and the curve $y = \sqrt{x}$ is revolved about the y-axis, the volume of the solid generated is

(A) $\frac{32\pi}{5}$ (B) $\frac{16\pi}{3}$ (C) $\frac{16\pi}{5}$

(D) $\frac{8\pi}{3}$ (E) π

Geometric Series,
nth Term Test
& Telescoping Series

Objective:

- Use the Geometric Series Test, nth Term Test, and Telescoping Test to determine if series converge.

If the sequence of partial sums converges, then the series is said to converge and has the sum indicated in the next definition.

Definitions of Convergent and Divergent Series

For the infinite series $\sum_{n=1}^{\infty} a_n$, the **nth partial sum** is

$$S_n = a_1 + a_2 + \cdot \cdot \cdot + a_n.$$

If the sequence of partial sums $\{S_n\}$ converges to S, then the series $\sum_{n=1}^{\infty} a_n$ **converges.** The limit S is called the **sum of the series.**

$$S = a_1 + a_2 + \cdot \cdot \cdot + a_n + \cdot \cdot \cdot \qquad S = \sum_{n=1}^{\infty} a_n$$

If $\{S_n\}$ diverges, then the series **diverges.**

Consider the series:

$$\sum_{n=1}^{\infty} \frac{1}{2^n}$$

List out the first four partial sums, then list out an explicit formula to represent the partial sums (S_n). Determine if the series converges or diverges based on your partial sums.

Geometric Series Test (GST)

A geometric series is in the form $\sum_{n=0}^{\infty} a \cdot r^n$ or $\sum_{n=1}^{\infty} a \cdot r^{n-1}$, $a \neq 0$

The geometric series **diverges** if $|r| \geq 1$.

If $|r| < 1$, the series **converges** to the sum $S = \frac{a_1}{1-r}$.

Where a_1 is the first term, regardless of where *n* starts, and *r* is the common ratio

Determine if the following series converge

(a) $\sum_{n=1}^{\infty} \frac{3}{2^n}$

(b) $\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n$

(c) $\sum_{n=2}^{\infty} 3\left(-\frac{1}{2}\right)^n$

_n_th Term Test for Divergence (ONLY)

If $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.

(think about it, it should make perfect sense!)

(a) $\sum_{n=1}^{\infty} \frac{2n+3}{3n-5}$

(b) $\sum_{n=1}^{\infty} \frac{n!}{2n!+1}$

(c) $\sum_{n=1}^{\infty} \frac{3^n - 2}{3^n}$

(d) $\sum_{n=2}^{\infty} \frac{1}{(1.1)^n}$

A series such as $\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\cdots$ is called a **telescoping series** because it collapses to one term or just a few terms. If a series collapses to a finite sum, then it converges by the **Telescoping Series Test**.

(a) $\sum_{n=1}^{\infty} \left(\frac{1}{2n+1} - \frac{1}{2n+3} \right)$

(c) $\sum_{n=1}^{\infty} \frac{1}{n^2 + 4n + 3}$