WARM UP - Calculator Inactive

1. Find the exact area of the region bounded by the graph of $f(x) = \sin x$, $g(x) = \cos x$, and the lines $y=0$ to $x = \pi/2$.

2. Find the equation of the tangent line for $f(x) = \sin x$ at $x = 0$, and then use it to approximate $\sin(0.1)$.

Taylor Polynomials

Objective:

- Find polynomial approximation of basic functions and compare with the actual function.
- Find Taylor and Maclaurin polynomials for basic functions.

The equation of the tangent line used in the warm up is called a **first-degree Taylor polynomial**.

Taylor polynomials of higher degree can be used to obtain increasingly better approximations of non-polynomial functions within a certain **radius** from a **center of approximation** x = c.

Example 1: (window X [-9,9], Y [-4,4])

Graph y1 = sinx

$$y2 : x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$$

adding an extra term each time

Definition of an nth-degree Taylor polynomial:

If f has n derivatives at $x = c$, then the polynomial

$$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \cdots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

is called the nth-degree Taylor polynomial for f centered at c, named after Brook Taylor, an English mathematician.

Note 1: A first-degree Taylor polynomial is a tangent line to f at c.

Note 2: $\frac{f^{(n)}(c)}{n!}$ **is the coefficient of the** $(x-c)^n$ **term**

If $c = 0$, then $P_n(x) = f(0) + f'(0)(x) + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n$ is called the nth-degree Maclaurin polynomial for f, named after Scottish mathematician, Colin Maclaurin.

Example 2:
Find the Maclaurin polynomial of degree n = 5 for f(x) = sin x. Then approximate the value of sin (0.1).

Example 3:
Find the Taylor polynomial of degree n=6 for $f(x) = \ln x$ at $c = 1$. Then use $P_6(x)$ to approximate ln (1.1)

Example 4:
Suppose that g is a function which has continuous derivatives, and that $g(2) = 3$, $g'(2) = -4$, $g''(2) = 7$, and $g^3(2) = -5$. Write the Taylor polynomial of degree 3 for g centered at 2.

Example 5

Let f be a function with derivatives of all orders and for which f(2) = 7. When n is odd, the nth derivative of f is 0. When n is even the nth derivative is

$$f^{(n)}(2) = \frac{(n-1)!}{3^n}$$

Write the sixth-degree Taylor polynomial for f about x=2.

Example 6:

Use a third-degree Taylor approximation of e^x for x near 0 to

$$\lim_{x \to 0} \frac{e^x - 1}{2x}$$

CLASSWORK:

Find the *nth* Maclaurin polynomial for each function.

1. $f(x) = e^{4x}$, n=4

2. $f(x) = \cos(2x)$, n = 6

Find the *nth* Taylor polynomial centered at c and then find the approximation using your answer.

3. $f(x) = x^3 - 3x^2 + 2$, n=3, c=3, f(3.2)

4. $f(x) = \ln x$, n=4, c=2, f(2.1)