Warm-up

1. The slope of the tangent line to the graph of $4x^2 + cx - 2e^y = -2$ at $x = 0$ is 4. Find the value of c.

2. The area of the region enclosed by the graph of $y = x^2 + 1$ and the line $y = 5$ is

(A) $\frac{14}{3}$ (B) $\frac{16}{3}$ (C) $\frac{28}{3}$

(D) $\frac{32}{3}$ (E) $8\pi$

# Parametric Equations Day 1

Objective:

- Find the first and second derivative of a parametric equation.
- Find parametric arc length.

# Review: Graph the parametric equation.

$$x = t^2 - 4 \text{ and } y = \frac{t}{2}, \; -2 \le t \le 3$$

**THEOREM 10.7 Parametric Form of the Derivative**

If a smooth curve $C$ is given by the equations

$$x = f(t) \quad \text{and} \quad y = g(t)$$

then the slope of $C$ at $(x, y)$ is

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}, \quad \frac{dx}{dt} \ne 0.$$

Because $dy/dx$ is a function of $t$, you can use Theorem 10.7 repeatedly to find *higher-order* derivatives. For instance,

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dt}\left[\frac{dy}{dx}\right] \cdot \frac{dt}{dx} = \frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{\frac{dx}{dt}}$$

Example 1: Find $dy/dx$ and $d^2y/dx^2$ and evaluate at $t=1$. $x = 2\sqrt{t}$ $y = 3t^2 - 2t$

Example 2: Given $x=4\cos t$ and $y = 3\sin t$, write an equation of the tangent line to the curve at the point where $t = 3\pi/4$.

Example 3: Find all points of horizontal and vertical tangency given $x = t^2 + t$ and $y = t^2 - 3t + 5$

**THEOREM 10.8 Arc Length in Parametric Form**

If a smooth curve $C$ is given by $x = f(t)$ and $y = g(t)$ such that $C$ does not intersect itself on the interval $a \le t \le b$ (except possibly at the endpoints), then the arc length of $C$ over the interval is given by

$$s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2}\, dt.$$

Example 4: Without a calculator, find the arc length of the given curve if

$$x = t^2, y = 4t^3 - 1, 0 \leq t \leq 1.$$

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6. A particle moves along the curve defined by the equation $y = x^3 - 3x$. The $x$–coordinate of the particle, $x(t)$, satisfies the equation $\frac{dx}{dt} = \frac{1}{\sqrt{2t+1}}$, for $t \geq 0$ with initial condition $x(0) = -4$.

(a) Find $x(t)$ in terms of $t$.

(b) Find $\frac{dy}{dt}$ in terms of $t$.

~~(c) Find the location and speed of the particle at time $t = 4$.~~

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|---|---|
| (a) $x(t) = \int \frac{1}{\sqrt{2t+1}}\,dt$<br>$x(t) = \sqrt{2t+1} + C$<br>$x(0) = -4 = 1 + C \implies C = -5$<br>$x(t) = \sqrt{2t+1} - 5$ | **3** { 1: $x(t) = \int \frac{dt}{\sqrt{2t+1}}$<br>1: $x(t) = \sqrt{2t+1} + C$<br>1: evaluates $C$ |
| (b) $y = x^3 - 3x$<br>$\frac{dy}{dt} = 3x^2\frac{dx}{dt} - 3\frac{dx}{dt}$<br>$= (3x^2 - 3)\frac{dx}{dt}$<br>$= \left[3\left(\sqrt{2t+1} - 5\right)^2 - 3\right]\left[\frac{1}{\sqrt{2t+1}}\right]$ | **2**: answer<br>< −1 > each error<br>Note: failure to express $\frac{dy}{dt}$ solely in terms of $t$ is a single error |