WARM UP:

1. If $y = \sec^2(x^2)$ find dy/dx.

2. In the xy-plane, the line $2x - y = 1$, where *k* is a constant, is the tangent to the graph of $y = k - x^2$. What is the value of k?

(A) -3 (B) -2 (C) -1 (D) 0 (E) 1

Implicit Differentiation

Objective:

- Distinguish between functions written in implicit and explicit form.
- Use implicit differentiation to find the derivative of an equation.

Implcit and Explicit Equations

Explicit - the equation is solve explicitly for one variable.

Ex: $y = 3x^2 - 6x + 5$ or $y = \tan x + \cos x$

Implicit - usually the x & y's are together on the same side.

Ex: $x^2 + y^2 = 36$ or $xy = x + y$

To understand implicit differentiation, you need to known with respect to what variable the differentiation is taking place.

$$\frac{d}{dx}[x^2] = 2x$$ Derivative with respect to x

If you derive another variable, you must involve the chain rule:

$$\frac{d}{dx}[y] = \frac{dy}{dx}$$

Variables agree: use Simple Power Rule.

$$\frac{d}{dx}[x^3] = 3x^2$$

Variables agree

Variables disagree: use Chain Rule.

$$\frac{d}{dx}[y^3] = 3y^2\frac{dy}{dx}$$

add dy/dx for derivative of y with respect to x

Variables disagree

GUIDELINES FOR IMPLICIT DIFFERENTIATION

1. Differentiate both sides of the equation *with respect to x*.
2. Collect all terms involving dy/dx on the left side of the equation and move all other terms to the right side of the equation.
3. Factor dy/dx out of the left side of the equation.
4. Solve for dy/dx.

Ex.1: Find dy/dx: $y^3 + y^2 - 5y - x^2 = -4$

Ex.2: Find dy/dx: $x^2 + xy + \cos(y) = 8y$

Ex.3: Evaluate dy/dx at the given point.

$$(x + y)^3 = x^3 + y^3, \quad (-1, 1)$$

Ex.4: Write the equation of the line tangent to the curve at the point y = 2

$$y + \sqrt{xy} = 4$$

Guided Practice - Classwork

FIND $\frac{dy}{dx}$ FOR EACH OF THE FOLLOWING.

(1) $x^2 - y^2 = 5$

(2) $1 - xy = x - y$

(3) $y^2 = x^3$

(4) $x = \tan y$

(5) $x^3 - xy + y^3 = 1$

(6) $9x^2 + 25y^2 = 225$

(7) Find the equation of both the tangent and normal lines to the curve $x^2 \cos^2 y - \sin y =$ (at the point $(0, \pi)$.

(8) Find the equation of both the tangent and normal lines to the curve $y^2(2 - x) = x^3$ a the point $(1, 1)$.

(9) Find $\frac{dy}{dx}$ for $x^2y + xy^2 = 2x$ at $(1,1)$

(10) Find $\frac{dy}{dx}$ for $(x+y)^2 + y = 2$ at $(0,1)$

WARM UP

1. Find the slope of the graph of $y^3 + y^2 - 5y - x^2 = -4$ at $(1,-3)$.

3. Find the equation of the tangent line to the graph of $f(x) = 2\sin x + \cos 2x$ at $x = \pi$.

Implicit Differentiation (Day 2)

Objective:

- Find first and second derivatives implicitly.
- Find horizontal and vertical tangent lines to a curve.

Ex.1: Given $x^2 + y^2 = 1$, find dy/dx and d^2y/dx^2.

Ex.2:Find d^2y/dx^2: $x^2y - 4x = 5$

Horizontal & Vertical Tangents

- Horizontal tangents occur when $\frac{dy}{dx} = \frac{0}{\neq 0}$
- Vertical tangents occur when $\frac{dy}{dx} = \frac{\neq 0}{0}$
- No tangent line exists when $\frac{dy}{dx} = \frac{0}{0}$ (these points must be thrown out)

Ex.3: Determine the x-value(s) of any horizontal tangent lines to the graph, $2x^2 + xy + 4y^2 = 3$.

Ex.4: Determine the y-value(s) of any vertical tangent lines to the graph, $2x^2 + xy + 4y^2 = 3$.

AP QUESTION

Find dy/dx: $y = \sqrt{x^2 + 2x - 1}$

A. $\frac{x+1}{y}$

B. $4y(x + 1)$

C. $\frac{1}{2\sqrt{x^2+2x-1}}$

D. $-\frac{x+1}{(x^2+2x-1)^{3/2}}$

E. none of these

Practice FRQ (first of many...)

Question 6

Consider the closed curve in the xy-plane given by

$$x^2 + 2x + y^4 + 4y = 5.$$

(a) Show that $\frac{dy}{dx} = \frac{-(x+1)}{2(y^3+1)}$.

(b) Write an equation for the line tangent to the curve at the point $(-2, 1)$.

(c) Find the coordinates of the two points on the curve where the line tangent to the curve is vertical.

(d) Is it possible for this curve to have a horizontal tangent at points where it intersects the x-axis? Explain your reasoning.

(a) $2x + 2 + 4y^3 \frac{dy}{dx} + 4\frac{dy}{dx} = 0$

$$\left(4y^3 + 4\right)\frac{dy}{dx} = -2x - 2$$

$$\frac{dy}{dx} = \frac{-2(x+1)}{4\left(y^3+1\right)} = \frac{-(x+1)}{2\left(y^3+1\right)}$$

$2 : \begin{cases} 1 : \text{implicit differentiation} \\ 1 : \text{verification} \end{cases}$

(b) $\left.\frac{dy}{dx}\right|_{(-2,\,1)} = \frac{-(-2+1)}{2(1+1)} = \frac{1}{4}$

Tangent line: $y = 1 + \frac{1}{4}(x+2)$

$2 : \begin{cases} 1 : \text{slope} \\ 1 : \text{tangent line equation} \end{cases}$

(c) Vertical tangent lines occur at points on the curve where $y^3 + 1 = 0$ (or $y = -1$) and $x \neq -1$.

On the curve, $y = -1$ implies that $x^2 + 2x + 1 - 4 = 5$, so $x = -4$ or $x = 2$.

Vertical tangent lines occur at the points $(-4, -1)$ and $(2, -1)$.

$3 : \begin{cases} 1 : y = -1 \\ 1 : \text{substitutes } y = -1 \text{ into the equation of the curve} \\ 1 : \text{answer} \end{cases}$

(d) Horizontal tangents occur at points on the curve where $x = -1$ and $y \neq -1$.

The curve crosses the x-axis where $y = 0$.

$$(-1)^2 + 2(-1) + 0^4 + 4 \cdot 0 \neq 5$$

No, the curve cannot have a horizontal tangent where it crosses the x-axis.

$2 : \begin{cases} 1 : \text{works with } x = -1 \text{ or } y = 0 \\ 1 : \text{answer with reason} \end{cases}$