WARM UP - No Calculator

1. If you were to use 3 midpoint rectangles of equal length to approximate the area under the curve of $f(x) = x^2 + 2$ from $x = 0$ to $x = 3$, how close would the approximation be to the exact area under the curve?

(a) 1/2 (b) 1/4 (c) 1/8 (d) 3/4 (e) 3/8

2. Evaluate $\int e^{(5x-1)}dx$

Arc Length

Objective:

- Find the arc length of smooth curves

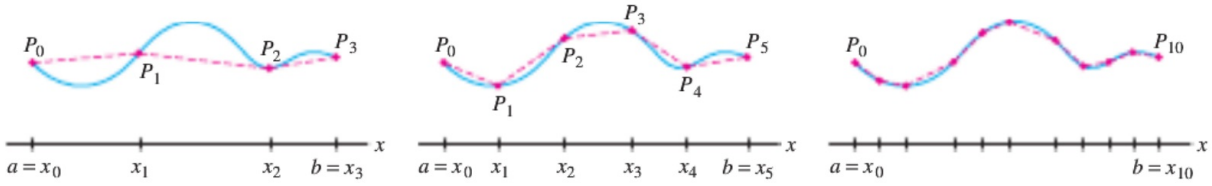
Definition of Arc Length

Let the function $y = f(x)$ represent a smooth curve on the interval $[a, b]$. The **arc length** of f between a and b is

$$s = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx.$$

Similarly, for a smooth curve $x = g(y)$, the **arc length** of g between c and d is

$$s = \int_c^d \sqrt{1 + [g'(y)]^2}\,dy.$$



$N = 3$ $N = 5$ $N = 10$

Ex. 1: Find the arc length of the graph of $f(x) = \ln(\sec x)$ from $x = 0$ to $x = \pi/4$. (calc)

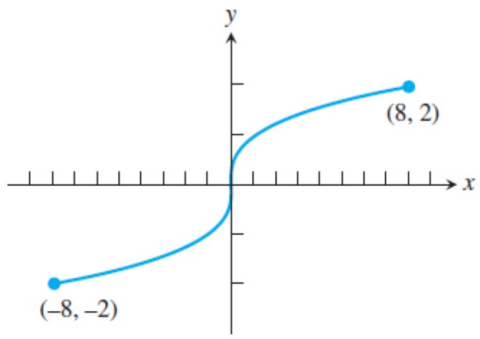
Ex. 2: Find the exact length of the curve (no calc)

$$y = \frac{4\sqrt{2}}{3}x^{3/2} - 1 \text{ for } 0 \le x \le 1.$$

Ex. 3: Determine the length of the curve (calc)

$x = \frac{2}{3}(y-1)^{3/2}$ between $1 \le y \le 4$

Ex.4: Find the length of the curve $y = x^{1/3}$ between (-8,-2) and (8,2) (calc)



5. Let R be the region enclosed by the graphs of $y = \ln(x^2 + 1)$ and $y = \cos x$. Write an expression involving one or more integrals that gives the length of the boundary of R. (calc)