

## WARM UP

1. Let R be the region in the first quadrant bounded by the x-axis and the curves  $y = \tan x$  and  $y = 2 - x^3$ . Find the area of region R. (calc)

2. (No calc) If  $f(x) = (x-1)^2 + \frac{e^{x-2}}{2}$ , then  $f'(2) =$

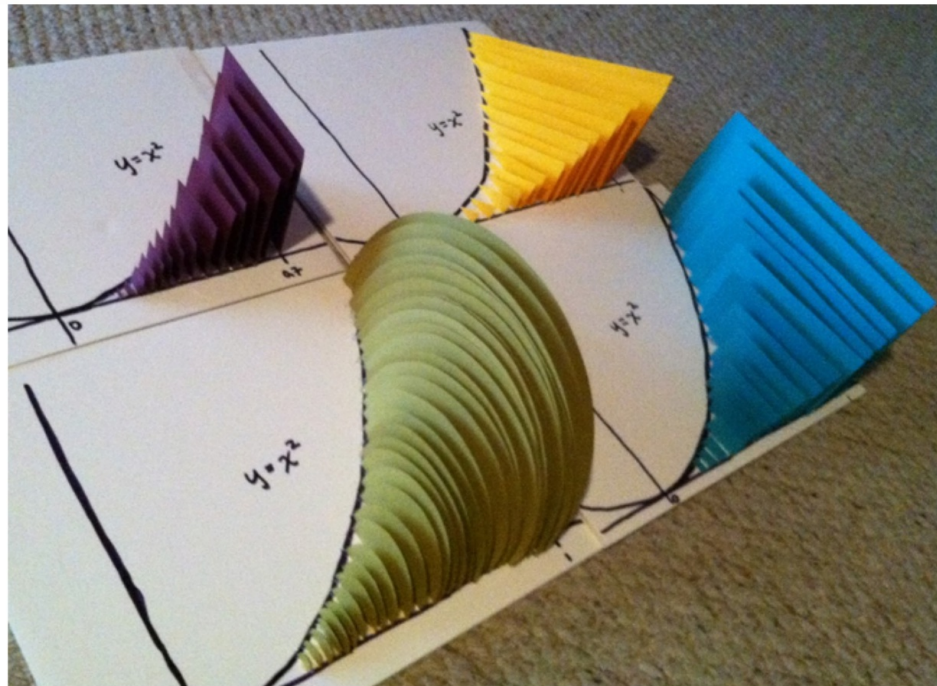
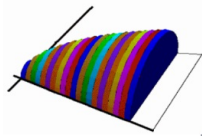
(A) 1                      (B)  $\frac{3}{2}$                       (C) 2

(D)  $\frac{7}{2}$                       (E)  $\frac{3+e}{2}$

## Cross Section Volumes

Objective:

- Find the volume of a solid using a known cross section.



- Cross sections perpendicular to  $x$ -axis:

$$V = \int_a^b A(x) dx$$

- Cross sections perpendicular to  $y$ -axis:

$$V = \int_c^d A(y) dy$$

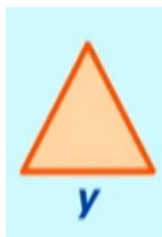
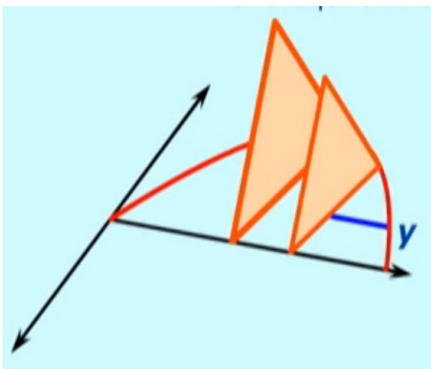
Where  $A(x)$  and  $A(y)$  is the area formula of your cross section in terms of  $x$  or  $y$ .

## Steps for Cross Section Problems

1. Draw figure. Check which axis the cross sections are perpendicular to.
2. Create the area using the base formulas.
3. The volume is the sum of all the areas.



Ex.1: Find the volume with the the figure created using the base  $y = \sin x$  over the interval  $[0, \pi]$  with cross sections of equilateral triangles, perpendicular to the x-axis.



Ex.2: The region  $R$ , in the first quadrant, is the base of a solid bounded by the graphs  $y = \sin \pi x$  and  $y = x - x^2$ . For this solid, each cross section is a semi-circle perpendicular to the  $x$ -axis. Find the volume.

Ex.3: Find the volume of the solid whose base is bounded by the graphs  $x = -y^2 + 2$  and  $y = x$ , with a cross section of an isosceles right triangle (base is a leg) perpendicular to the  $y$ -axis. (No Calculator for set up)

Ex.4: Find the volume of the solid whose base is bounded by the graphs  $x = -y^2 + 2$  and  $y = x$ , with a cross section of a rectangle whose height is 3 times the base and perpendicular to the  $y$ -axis.

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**Question 1**

Let  $R$  be the shaded region bounded by the graphs of  $y = \sqrt{x}$  and  $y = e^{-3x}$  and the vertical line  $x = 1$ , as shown in the figure above.

- Find the area of  $R$ .
- Find the volume of the solid generated when  $R$  is revolved about the horizontal line  $y = 1$ .
- The region  $R$  is the base of a solid. For this solid, each cross section perpendicular to the  $x$ -axis is a rectangle whose height is 5 times the length of its base in region  $R$ . Find the volume of this solid.

