WARM UP - No Calculator

What is the area of the region in the first quadrant bounded by the graph of $y = e^{x/2}$ and the line $x = 2$?

(A) $2e - 2$ (B) $2e$ (C) $\frac{e}{2} - 1$ (D) $\frac{e-1}{2}$ (E) $e - 1$

A particle moves along the x-axis with velocity given by $v(t) = 3t^2 - 4$ for time $t \geq 0$. If the particle is at position $x = -2$ at time $t = 0$, what is the position of the particle at time $t = 3$?

(A) 13 (B) 15 (C) 16 (D) 17 (E) 25

$\int_{-2}^{2} \left(x^7 + k\right) dx = 16$, then $k =$

(A) -12 (B) -4 (C) 0 (D) 4 (E) 12

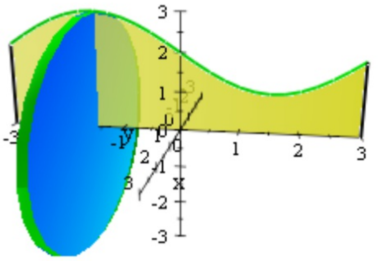
Volume of Revolutions
Shell Method

Objective:

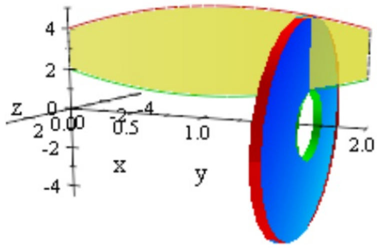
- Find the volume of a plane rotated around an axis using the shell method.

Volumes of Revolution Review

Disk method

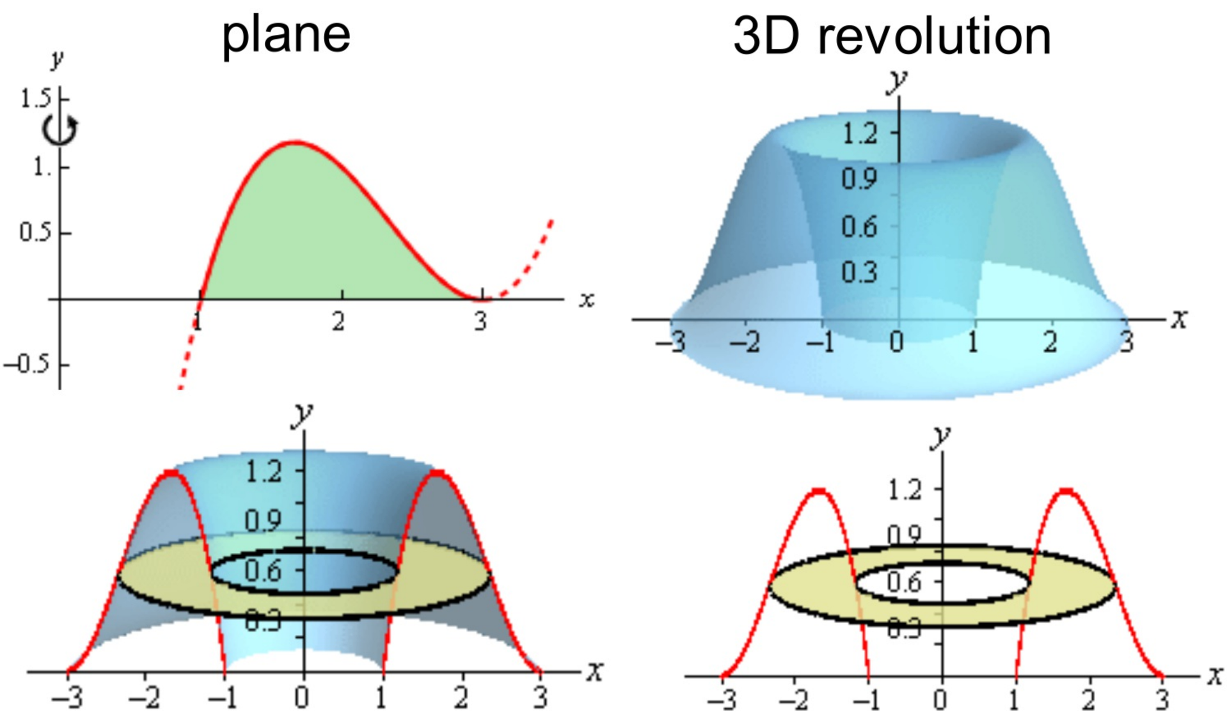


Washer Method



The cross sections are perp. to the axis of rotation.

Example 1 - Calc
Determine the volume of the solid obtained by rotating the region bounded by $y = (x-1)(x-3)^2$ and the x-axis about the y-axis.



What are the problems using the washer method?

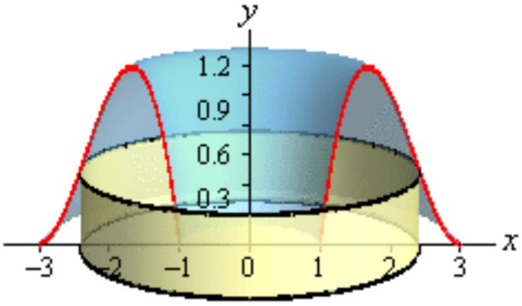
1. Both outer and inner radii (R and r) are defined by the equation $y = (x - 1)(x - 3)^2$.

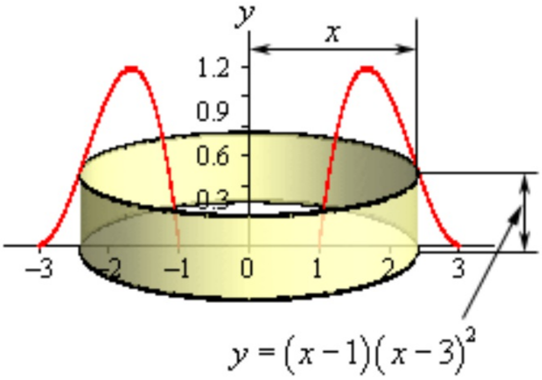
2. The equation needed for y-axis rotation is in terms of y (nearly impossible).

3. We will need to find a maximum by hand to define where 'R' becomes 'r'.

SHELL METHOD

This method uses the radius in terms of x.





This creates a cylinder or "shell" in the figure.

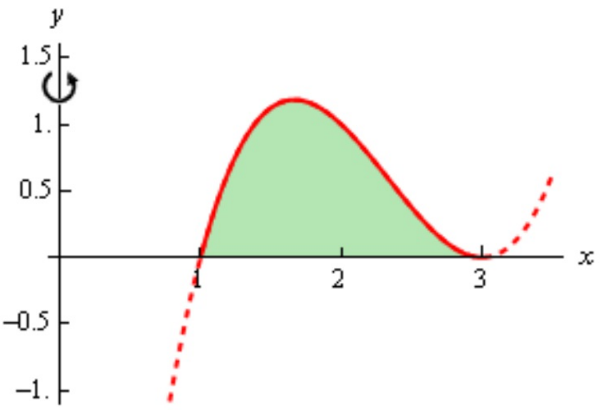
Note - the cross sectons are parallel to the axis of rotation.

Surface area of the shell

SA = 2π(radius)(height)

radius - r(x) and height - h(x)

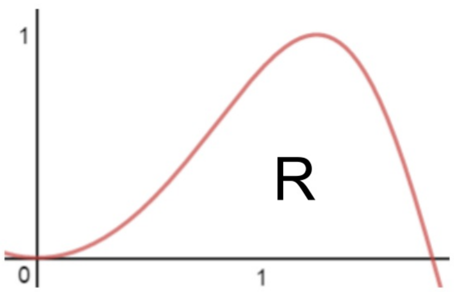
$$V = 2\pi \int_a^b r(x) \cdot h(x)dx$$



$y = (x-1)(x-3)^2$

Ex.2: No Calc

Determine the volume of the solid obtained by rotating the region R bounded by the curve $y = \sin(x^2)$ and the x-axis about the y-axis.



Ex.3: No Calc

Rotate the region bounded by $x = (y - 2)^2$, the x-axis and the y-axis about the x-axis.

Ex.4: No Calc

Rotate the region in the first quadrant bounded by $y = 4x$ and $y = x^3$ about the y-axis.

Ex.5: No Calc

Determine the volume of the solid obtained by rotating the region bounded by $y = x^{1/2}$ and $y = x$ around $x = -4$ (by Washer & Shell)